

Question 1 [15 marks]

Solve the following inequalities and write the solution in interval form:

$$\frac{1}{2} + 3x < \frac{1}{3} - 2x \leq \frac{1}{3}x + 5.$$

$$2 \times 3 = 6$$

$$6 \times \frac{1}{2} + 6 \times 3x < 6 \times \frac{1}{3} - 6 \times 2x \leq 6 \times \frac{1}{3}x + 6 \times 5$$

$$3 + 18x < 2 - 12x \leq 2x + 30$$

$$3 + 18x < 2 - 12x$$

$$18x + 12x < 2 - 3$$

$$\frac{30x}{30} < \frac{-1}{30}$$

$$x < -\frac{1}{30}$$

$$2 - 12x \leq 2x + 30$$

$$2 - 30 \leq 2x + 12x$$

$$\frac{-28}{14} \leq \frac{14x}{14}$$

$$-2 \leq x$$

graph



$$x \in [-2, -\frac{1}{30})$$

solution in interval form

Question 2 [10 marks]

Solve the following system using elimination by addition:

$$3x - 2y = 4$$

$$2x + 3y = 7$$

$$-2 \times \quad 3x - 2y = 4 \quad - (1)$$

$$3 \times \quad 2x + 3y = 7 \quad - (2)$$

$$\begin{array}{r} -6x + 4y = -8 \\ -6x + 9y = 21 \end{array} \quad \left. \vphantom{\begin{array}{r} -6x + 4y = -8 \\ -6x + 9y = 21 \end{array}} \right\} \text{عكس!}$$

$$\begin{array}{r} 13y = +13 \\ \hline 13 \end{array}$$

$$\boxed{y = 1}$$

$$3x - 2y = 4$$

$$3x - 2(1) = 4$$

$$3x - 2 = 4$$

$$3x = 4 + 2$$

$$\begin{array}{r} 3x = 6 \\ \hline 3 \end{array}$$

$$\boxed{x = 2}$$

Solution $x = 2$, $y = 1$

Question 3 [15 marks]

(a) Find an equation of the line that passes through $(0, 2)$ and $(1, 0)$.

(b) Find an equation of the line that has slope 2 and passes through $(-6, 0)$.

(c) Find the intersection of the two lines given in parts (a) and (b).

(a) $y = mx + b$ $\rightarrow (y - y_1) = m(x - x_1)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 2}{1 - 0} = \frac{-2}{1} = -2$$

$$(y - 2) = -2(x - 0)$$
$$y - 2 = -2x$$

$$y = -2x + 2$$

(b) $(y - y_1) = m(x - x_1)$

$$m = 2$$

$$(x_1, y_1) = (-6, 0)$$

$$(y - 0) = 2(x - (-6))$$

$$y = 2(x + 6)$$

$$y = 2x + 12$$

(c) intersection of two lines :-

$$y_1 = -2x + 2$$

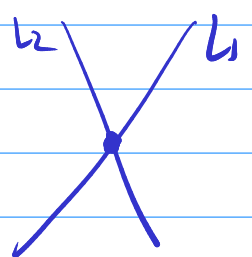
$$y_2 = 2x + 12$$

$$y_1 = y_2$$
$$-2x + 2 = 2x + 12$$

$$-12 + 2 = 2x + 2x$$

$$\frac{4x}{4} = -\frac{10}{4}$$

$$x = -\frac{5}{2}$$



$$y = -2x + 2 = -2(-2.5) + 2 = +5 + 2$$

intersection point $(-2.5, 7)$

$$\boxed{y = 7}$$

Question 4 [20 marks]

Solve the following system using Gauss-Jordan elimination:

$$3x_1 + 8x_2 - x_3 = -18$$

$$2x_1 + x_2 + 5x_3 = 8$$

$$2x_1 + 4x_2 + 2x_3 = -4$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right] \text{ النوع }$$

augmented matrix

$$\left[\begin{array}{ccc|c} 3 & 8 & -1 & -18 \\ 2 & 1 & 5 & 8 \\ 2 & 4 & 2 & -4 \end{array} \right]$$

$$R_1 \leftrightarrow R_3$$

$$\left[\begin{array}{ccc|c} 2 & 4 & 2 & -4 \\ 2 & 1 & 5 & 8 \\ 3 & 8 & -1 & -18 \end{array} \right]$$

$$R_1 / 2$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & -2 \\ 2 & 1 & 5 & 8 \\ 3 & 8 & -1 & -18 \end{array} \right]$$

$$\begin{array}{l} -2R_1 + R_2 \\ -3R_1 + R_3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & -2 \\ 0 & -3 & 3 & 12 \\ 0 & 2 & -4 & 12 \end{array} \right]$$

$$R_2 / -3$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & -2 \\ 0 & 1 & -1 & -4 \\ 0 & 2 & -4 & 12 \end{array} \right]$$

$$\begin{array}{l} -2R_2 + R_1 \\ -2R_2 + R_3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 6 \\ 0 & 1 & -1 & -4 \\ 0 & 0 & -2 & 20 \end{array} \right]$$

$$R_3 / -2$$

$$\begin{array}{c} \textcircled{-3} \rightarrow \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 3 & 6 \\ 0 & 1 & -1 & -4 \\ 0 & 0 & 1 & -10 \end{array} \right] \quad \begin{array}{l} R_3 + R_2 \\ \hline -3R_3 + R_1 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 36 \\ 0 & 1 & 0 & -14 \\ 0 & 0 & 1 & -10 \end{array} \right]$$

solution

$$X_1 = 36$$

$$X_2 = -14$$

$$X_3 = -10$$

Question 5 [10 marks]

Write the linear inequalities defining the shaded region:

Line ①

(0, 6) (6, 0)

$$y = mx + b$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{0 - 6}{6 - 0} = (-1)$$

$$y = -x + b$$

$$\boxed{6 = b}$$

$$y \leftarrow y = -x + 6$$

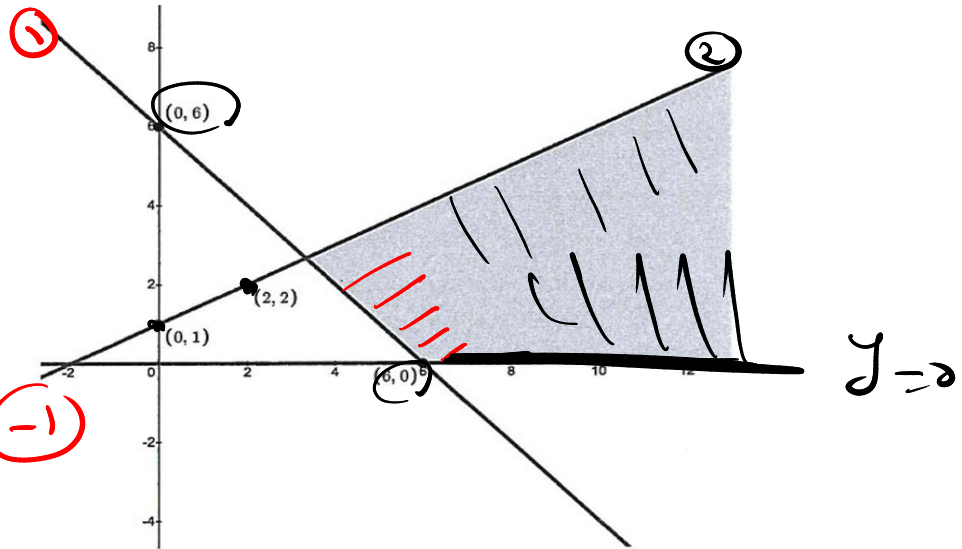
$$\boxed{y + x = 6}$$

$$\boxed{y + x \geq 6} \text{ --- ①}$$

$$y + x \geq 6 \text{ --- ①}$$

$$y - \frac{1}{2}x \leq 1 \text{ --- ②}$$

$$y \geq 0 \text{ --- ③}$$



② (0, 1) (2, 2)

$$y = mx + b$$

$$m = \frac{2 - 1}{2 - 0} = \left(\frac{1}{2}\right)$$

$$m = \frac{1}{2}$$

$$y = \frac{1}{2}x + b$$

$$y = 1 \quad x = 0$$

$$\boxed{b = 1}$$

$$y = \frac{1}{2}x + 1$$

$$\boxed{y - \frac{1}{2}x \leq 1}$$

Question 6 [15 marks]

Maximize and minimize $P = 4x + y$

subject to

$$3x + 2y \geq 18$$

$$x \leq 9$$

$$y \leq 6$$

$$x \geq 0, y \geq 0$$

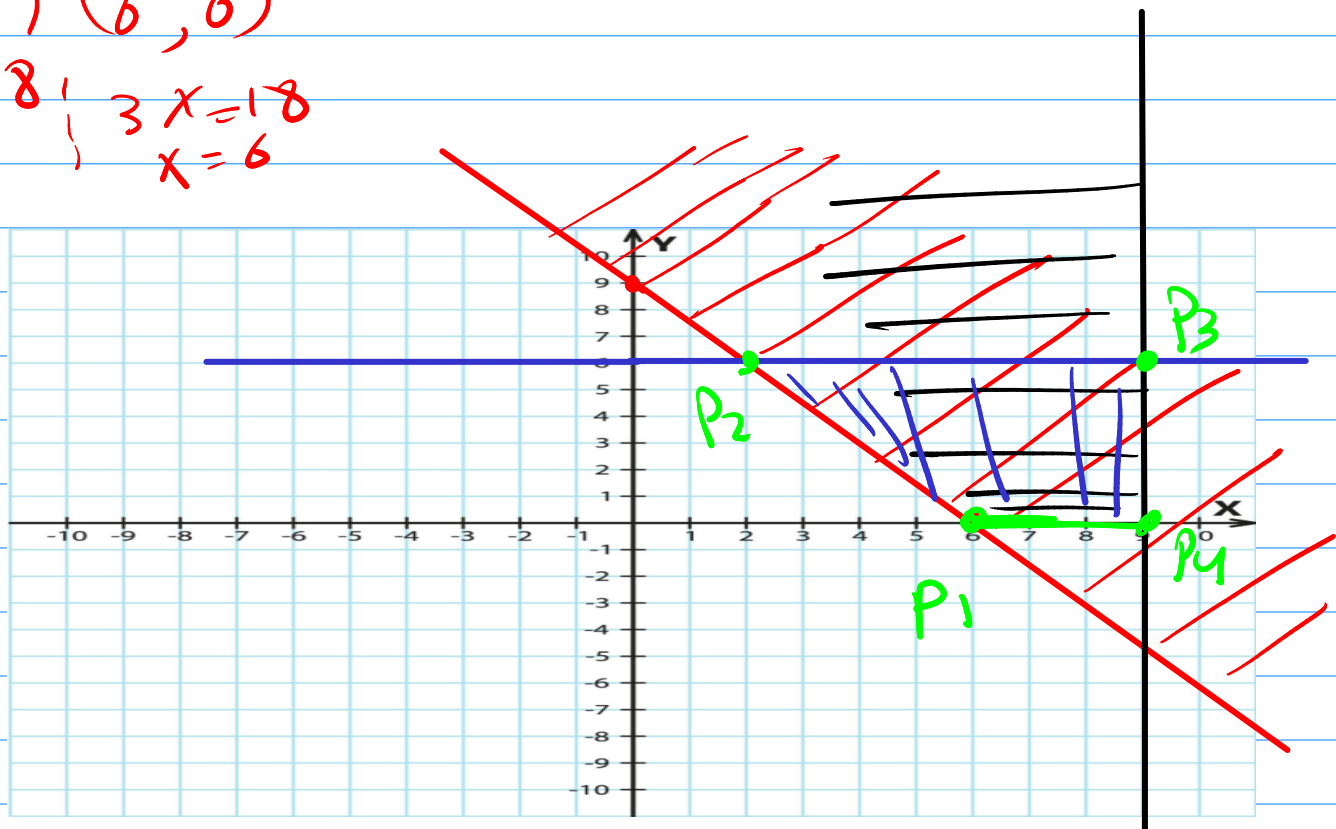
① $3x + 2y = 18$

② $x = 9$

③ $y = 6$

$(0, 9)$ $(6, 0)$

$2y = 18 \Rightarrow y = 9$
 $3x = 18 \Rightarrow x = 6$



bounded region $* P = 4x + y$

$P_1 = (6, 0) \rightarrow P = (6)(4) + 0 = 24$

$P_2 = (2, 6) \rightarrow P = 2(4) + 6 = 14$

$P_3 = (9, 6)$

$P = 42$

$P_4 = (9, 0)$

$P = 36$

$$\begin{array}{ll} \text{max } P = 42 & \text{at } x=9 \quad y=6 \\ \text{min } P = 14 & \text{at } x=2 \quad y=6 \end{array}$$

Question 7 [15 marks]

A chemical manufacturer wants to lease a fleet of 10 railroad tank cars with a combined carrying capacity of 45000 gallons. Tank cars with three different carrying capacities are available: 3000 gallons, 4000 gallons, and 5000 gallons. How many of each type of tank car should be leased?

x = Number of Tank Cars of 3000 gallons
 y = Number of Tank Cars of 4000 gallons
 z = Number of Tank Cars of 5000 gallons

$$x + y + z = 10 \quad x \geq 0 \quad y \geq 0 \quad z \geq 0$$

$$3000x + 4000y + 5000z = 45000$$

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$$3x + 4y + 5z = 45$$

$$(-3) \begin{bmatrix} 1 & 1 & 1 & 10 \\ 3 & 4 & 5 & 45 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad -3R_1 + R_2$$

$$\begin{bmatrix} 1 & 1 & 1 & 10 \\ 0 & 1 & 2 & 15 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad -1R_2 + R_1$$

$$\rightarrow \begin{bmatrix} 1 & 0 & -1 & -5 \\ 0 & 1 & 2 & 15 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$y = z$$

$$1y + 2z = 15$$

$$2z = 15 - z$$

$$\boxed{z = \frac{15 - t}{2}}$$

$$x - z = -5$$

$$x = -5 + z$$

$$x = -5 + \left(\frac{15 - t}{2}\right)$$

$$x = -\frac{5}{1} \times \frac{15}{2} - \frac{t}{2}$$

$$x = \frac{-10 + 15}{2} - \frac{t}{2}$$

$$\boxed{x = \frac{5}{2} - \frac{t}{2}}$$

* Solution

$$* x = \frac{5}{2} - \frac{t}{2}$$

$$\text{at } t = 3$$

$$* y = t$$

$$x = \frac{5}{2} - \frac{3}{2} = \frac{2}{2} = 1$$

$$* z = \frac{15 - t}{2}$$

$$y = 3$$

$$z = \frac{15 - 3}{2} = 6$$

$$\boxed{x = 1}$$

$$\boxed{y = 3}$$

$$\boxed{z = 6}$$