Question 1 [15 marks]

Solve and graph:

$$-7(x-2) \leq 2(x+5)+35$$

$$-7(x-2) \leq 2(x+5)+35$$

$$-7x+14 \leq 2x+10+35$$

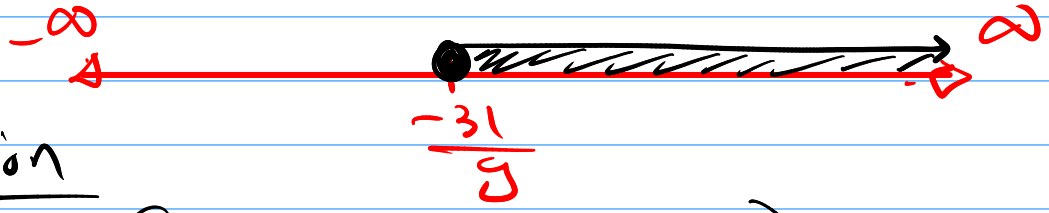
$$-7x+14 \leq 2x+45$$

$$14-45 \leq 2x+7x$$

$$-31 \leq 9x$$

$$\frac{-31}{9} \leq x$$

graph



Solution

$$x \in \left[\frac{-31}{9}, \infty\right)$$

Question 2 [10 marks]

If a cardboard box has length L, width W, and height H, then its surface area is given by the formula $S=2LW+2LH+2HW$. Solve the formula for L in terms of S, W, and H.

$$S = 2Lw + 2LH + 2HW$$

$$L = \text{common}$$

$$S - 2HW = 2Lw + 2LH$$

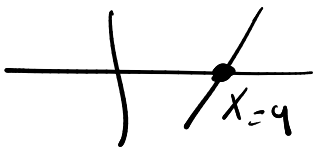
$$\frac{S - 2HW}{(2w+2H)} = \frac{L[2w+2H]}{(2w+2H)}$$

$$\boxed{L = \frac{S - 2HW}{2w+2H}}$$

Question 3 [15 marks]

Let L be the line defined by the equation $y=mx+b$. Find m and b if the line L passes through the point $(2,6)$ and intersects the x-axis at $x=4$. $(y=0)$

$y = mx + b$



$P(2,6)$ (x_1, y_1)

$(4,0)$ (x_2, y_2)

x intercept at $x=4$

$x=4$ $y=0$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-6}{4-2}$$

$$m = \frac{-6}{2} = \boxed{-3}$$

$y = -3x + b$

$P(2,6)$ $x=2$ $y=6$

$6 = -3(2) + b$

$6 = -6 + b$

$6 + 6 = b$

$\boxed{b = 12}$

$\boxed{y = -3x + 12}$

Question 4 [20 marks]

Solve by Gauss-Jordan elimination the following system:

$$3x+2y-5z=4$$
$$x-y-3z=3$$
$$4x+y-8z=11$$

augmented matrix

$$\left[\begin{array}{ccc|c} 3 & 2 & -5 & 4 \\ 1 & -1 & -3 & 3 \\ 4 & 1 & -8 & 11 \end{array}\right]$$

$R_2 \leftrightarrow R_1$

$$\left[\begin{array}{ccc|c} 1 & -1 & -3 & 3 \\ 3 & 2 & -5 & 4 \\ 4 & 1 & -8 & 11 \end{array}\right]$$

$-3R_1+R_2$

$-4R_1+R_3$

$$\left[\begin{array}{ccc|c} 1 & -1 & -3 & 3 \\ 0 & 5 & 4 & -5 \\ 0 & 5 & 4 & -1 \end{array}\right]$$

$-4(1)+4=0$

$-4(-1)+1=$

$-4(-3)+-8$

$12+-8=$

$-4(3)+11=-1$

$-1R_2+R_3$

$$\left[\begin{array}{ccc|c} 1 & -1 & -3 & 3 \\ 0 & 5 & 4 & -5 \\ 0 & 0 & 0 & 4 \end{array}\right]$$

$0 \neq 4$

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There is no solution.

Question 5 [10 marks]

Solve the following system of linear inequalities graphically:

$$x+2y<8$$
$$2x-y\geq 4$$

$x + 2y = 8$

$(0, 4)$ $(8, 0)$

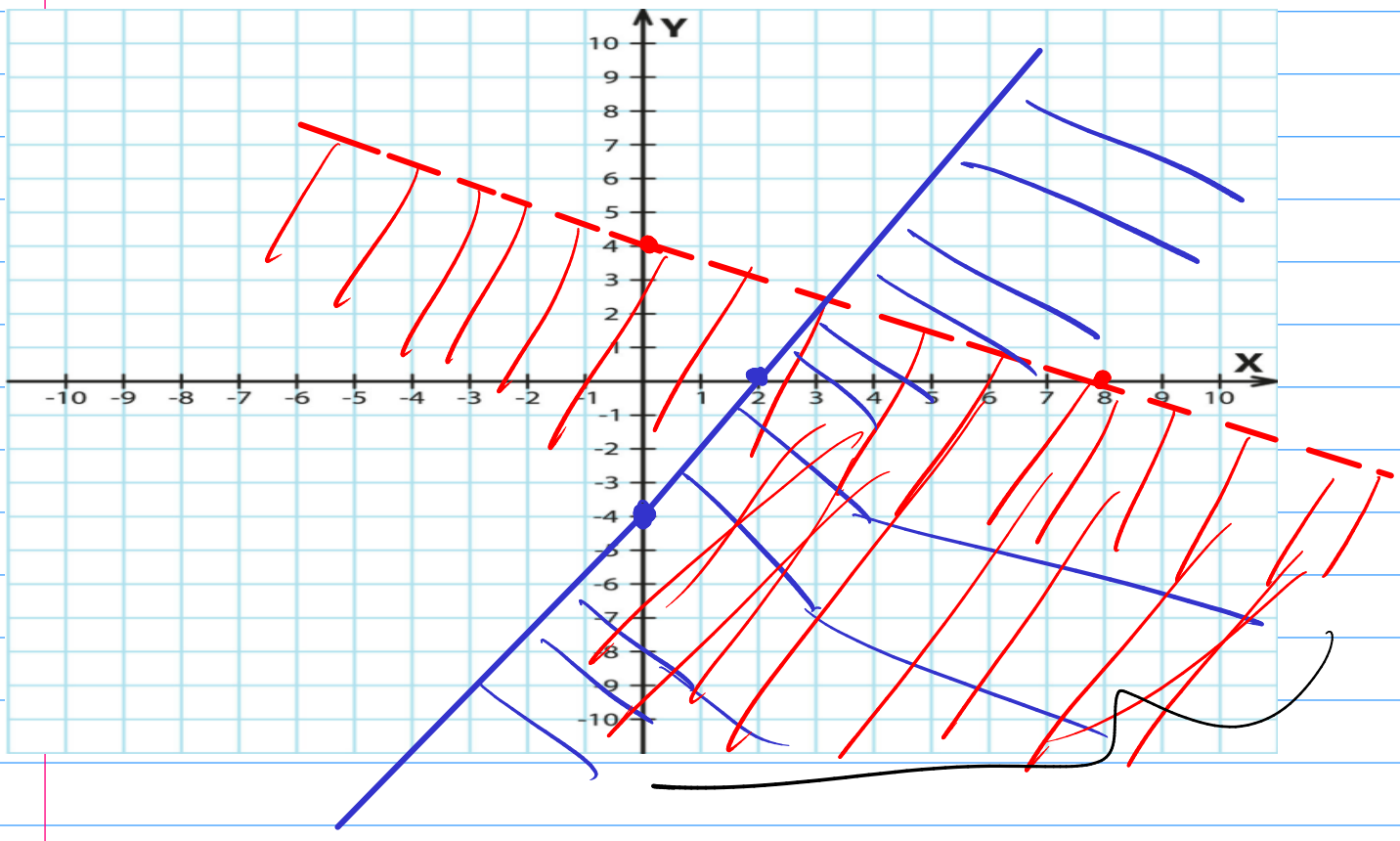
$\frac{2y}{2} = \frac{8}{2}$ $x = 8$

$2x - y = 4$

$(0, -4)$ $(2, 0)$

$-y = 4$ | $\frac{2x}{2} = \frac{4}{2}$

$y = -4$ | $x = 2$



Solution region

unbounded region

Question 6 [15 marks]

Minimize and Maximize the objective function: $P = 5x + 4y$, subject to

$$4x + 5y \leq 33$$
$$-2x + y \leq 1$$
$$x + 3y \geq 10$$

(1) $4x + 5y = 33$

$(0, \frac{33}{5})$ $(\frac{33}{4}, 0)$

$5y = 33$ | $4x = 33$

$y = \frac{33}{5}$ | $x = \frac{33}{4}$

$-2x + y = 1$

$(0, 1)$ $(-\frac{1}{2}, 0)$

$-2x = 1$

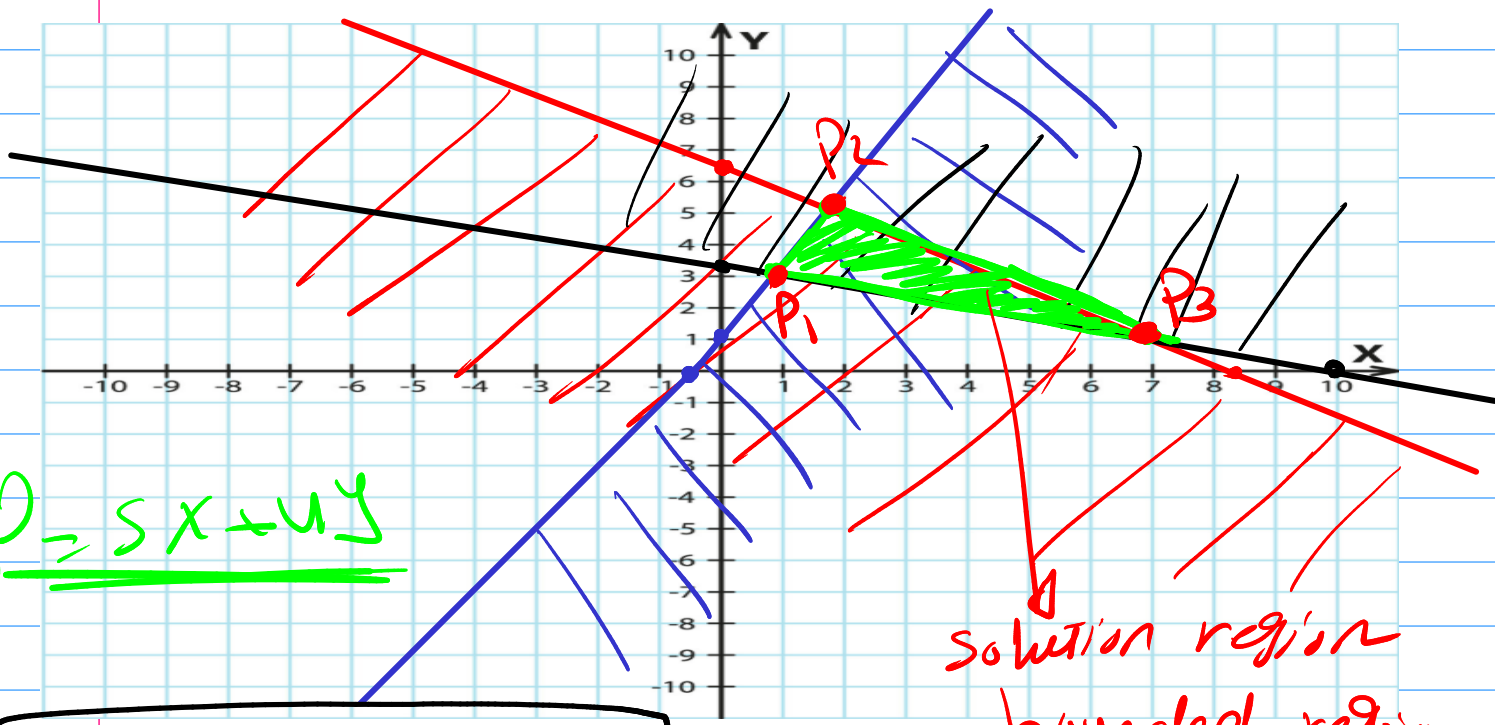
$x = -\frac{1}{2}$

$x + 3y = 10$

$(0, \frac{10}{3})$ $(10, 0)$

$3y = 10$

$y = \frac{10}{3}$



$P = 5x + 4y$

solution region

bounded region

$P_1 = (1, 3) \rightarrow P = 17$

$P_2 = (2, 5) \rightarrow P = 30$

$P_3 = (7, 1) \quad P = 39$

$-2x + y = 1$

$2 * \quad x + 3y = 10$

$-2x + 6y = 20$

$7y = 21$

$y = 3$

$$-2x + 3y = 1$$

$$-2x = 1 - 3$$

$$\frac{-2x}{-2} = \frac{-2}{-2}$$

$$\boxed{x = 1}$$

$$4x + 5y = 33$$

$$-2x + y = 1 \quad \rightarrow \times 2$$

$$-4x + 2y = 2$$

$$\frac{7y}{7} = \frac{35}{7}$$

$$\boxed{y = 5}$$

$$\therefore -2x + y = 1$$

$$-2x + 5 = 1$$

$$-2x = 1 - 5$$

$$\frac{-2x}{-2} = \frac{-4}{-2}$$

$$\boxed{x = 2}$$

$$-4x + 3y = 10 \quad (1)$$

$$4x + 5y = 33$$

$$-4x - 12y = -40$$

$$\frac{-7y}{-7} = \frac{-7}{-7}$$

$$\boxed{y = 1}$$

$$x + 3(1) = 10$$

$$x = 10 - 3$$

$$\boxed{x = 7}$$

Max $P = 35$ at $x = 7, y = 1$

. min $P = 17$ at $x = 1$, $y = 3$

Question 7 [15 marks]

A company that rents small moving trucks wants to purchase 18 trucks with a combined capacity of 19,200 cubic feet. Three different types of trucks are available: a small size truck with a capacity of 300 cubic feet, a medium size truck with a capacity of 600 cubic feet, and a big size truck with a capacity of 1,200 cubic feet. How many of each type of truck should the company purchase?

$x \longrightarrow$ number of trucks of small size
$y \longrightarrow$ number of trucks of medium size
$z \longrightarrow$ number of trucks of large size

$$x + y + z = 18$$

$$300x + 600y + 1200z = 19200$$

$$x \geq 0 \quad y \geq 0 \quad z \geq 0$$

$\div 300$

$$x + 2y + 4z = 64$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 18 \\ 1 & 2 & 4 & 64 \\ 0 & 0 & 0 & 0 \end{bmatrix} -1R_1 + R_2$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 18 \\ 0 & 1 & 3 & 46 \\ 0 & 0 & 0 & 0 \end{bmatrix} -1R_2 + R_1$$

$$\rightarrow \begin{matrix} x & y & z \end{matrix} \begin{bmatrix} 1 & 0 & -3 & -28 \\ 0 & 1 & 3 & 46 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$z = t$

$$y + 3z = 46$$

$$y + 3t = 46$$

$$y = 46 - 3gt$$

$$x - 38(t) = -28$$

$$x - 38t = -28$$

$$x = -28 + 38t$$

$$x = -28 + 38t$$

$$y = 46 - 3gt$$

$$z = t$$

at $t = 1$

$z = 1$

$y = 7$

$x = 10$