

$$5 \times 3 \times 2 = \boxed{30}$$

## Written Questions — 100 marks

### Question 1 [15 marks]

Solve and graph:  $\frac{2x-1}{5} - \frac{1-x}{3} < 2x-1 \leq \frac{3x+5}{2}$

$$\begin{aligned} & \textcircled{30} \times \frac{2x-1}{5} - \textcircled{30} \times \frac{1-x}{3} < \textcircled{30} (2x-1) \leq \textcircled{30} \times \frac{3x+5}{2} \\ & 6(2x-1) - 10(1-x) < 30(2x-1) \leq 15(3x+5) \end{aligned}$$

$$12x - 6 - 10 + 10x < 60x - 30 \leq 45x + 75$$

$$22x - 16 < 60x - 30 \leq 45x + 75$$

$$22x - 16 < 60x - 30$$

$$-16 + 30 < 60x - 22x$$

$$\frac{14}{38} < \frac{38x}{38}$$

$$\boxed{\frac{7}{19} < x}$$

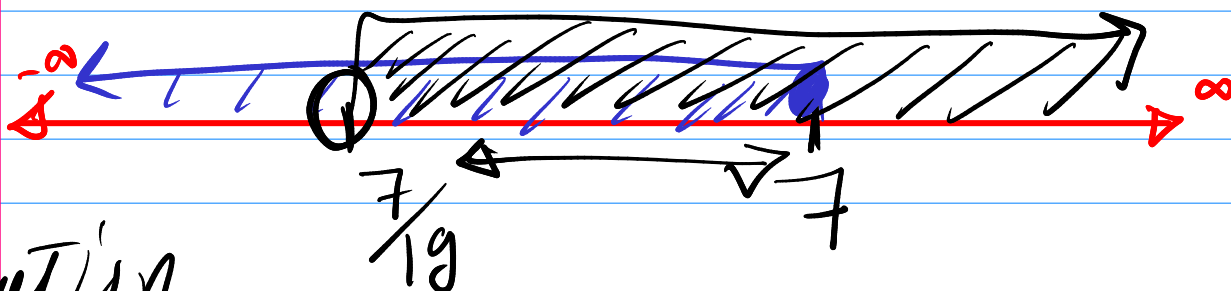
$$60x - 30 \leq 45x + 75$$

$$60x - 45x \leq 75 + 30$$

$$\frac{15x}{15} \leq \frac{105}{15}$$

$$\boxed{x \leq 7}$$

graph



Solution

$$X \in \left( \frac{7}{19}, 7 \right]$$

Question 2 [10 marks]

Solve the formula  $2r = \frac{5t+7r}{3t+1}$  for  $r$  in terms of  $t$ .

$r = \frac{5t}{6t-5}$

$$\cancel{2r = \frac{5t+7r}{3t+1}}$$

$$2r(3t+1) = 5t + 7r$$

$$2r(3t+1) - 7r = 5t$$

$$r[2(3t+1) - 7] = 5t$$

$$r[6t + \underline{2-7}] = 5t$$

$$r[6t - \underline{5}] = 5t$$

$$r = \frac{5t}{6t-5}$$

**Question 3** [15 marks]

A line  $y = mx + b$  passes through the points  $(2, 0)$  and  $(0, 3)$ . Find  $m$  and  $b$ .

$$P_1 = (2, 0)$$

$x_1$        $y_1$

$$P_2 = (0, 3)$$

$x_2$        $y_2$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{3 - 0}{0 - 2} = \boxed{\frac{3}{-2}}$$

$$y = mx + b$$

$$y = \frac{3}{-2}x + b \quad \text{at } (2, 0)$$

$$x = 2 \qquad y = 0$$

$$0 = \frac{3}{-2}(2) + b$$

$$0 = -3 + b$$

$$\boxed{b = 3}$$

$$\boxed{y = -\frac{3}{2}x + 3}$$

$$\boxed{\frac{3}{-2} = \frac{-3}{2} = -\frac{3}{2}}$$

#### Question 4 [20 marks]

Solve the following linear system using Gauss-Jordan elimination.

$$16x + 17y + 18z + 19w = 20$$

$$11x + 12y + 13z + 14w = 15$$

$$6x + 7y + 8z + 9w = 10$$

$$x + 2y + 3z + 4w = 5$$

\*augmented matrix

المطلوب  $\Rightarrow$   $\left[ \begin{array}{cccc|c} 1 & 0 & 0 & 0 & 5 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 4 \end{array} \right]$  التوابع

$$\left[ \begin{array}{cccc|c} 16 & 17 & 18 & 19 & 20 \\ 11 & 12 & 13 & 14 & 15 \\ 6 & 7 & 8 & 9 & 10 \\ 1 & 2 & 3 & 4 & 5 \end{array} \right]$$

$$R_1 \leftarrow R_4$$

$$\left[ \begin{array}{cccc|c} 1 & 2 & 3 & 4 & 5 \\ 6 & 7 & 8 & 9 & 10 \\ 11 & 12 & 13 & 14 & 15 \\ 16 & 17 & 18 & 19 & 20 \end{array} \right] \begin{array}{l} -6R_1 + R_2 \\ -11R_1 + R_3 \\ -16R_1 + R_4 \end{array}$$

$$\left[ \begin{array}{cccc|c} 1 & 2 & 3 & 4 & 5 \\ 0 & -5 & -10 & -15 & -20 \\ 0 & -10 & -20 & -30 & -40 \\ 0 & -15 & -30 & -45 & -60 \end{array} \right] \begin{array}{l} -32 + 17 = \\ -48 + 18 = \end{array}$$

$$R_2 \times 5 \rightarrow \left[ \begin{array}{cccc|c} 1 & 2 & 3 & 4 & 5 \\ 0 & 1 & 2 & 3 & 4 \\ 0 & -10 & -20 & -30 & -40 \\ 0 & -15 & -30 & -45 & -60 \end{array} \right] \begin{array}{l} 10R_2 + R_3 \\ 15R_2 + R_4 \\ -2R_2 + R_1 \end{array}$$

$$\left[ \begin{array}{cccc|c} 1 & 0 & -1 & -2 & -3 \\ 0 & 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

in finding many solution  
وجود عدد از شرطی ← هر کمال

$$y = z$$

$$z = 5$$

$$y + 2z + 3w = 4$$

$$t + 25 + 3w = 4$$

$$3w = 4 - t - 25$$

$$w = \frac{4 - t - 25}{3}$$

$$x - z - 2w = -3$$

$$x = -3 + z + 2w$$

$$x = -3 + 5 + 2 \left[ \frac{4 - t - 25}{3} \right]$$

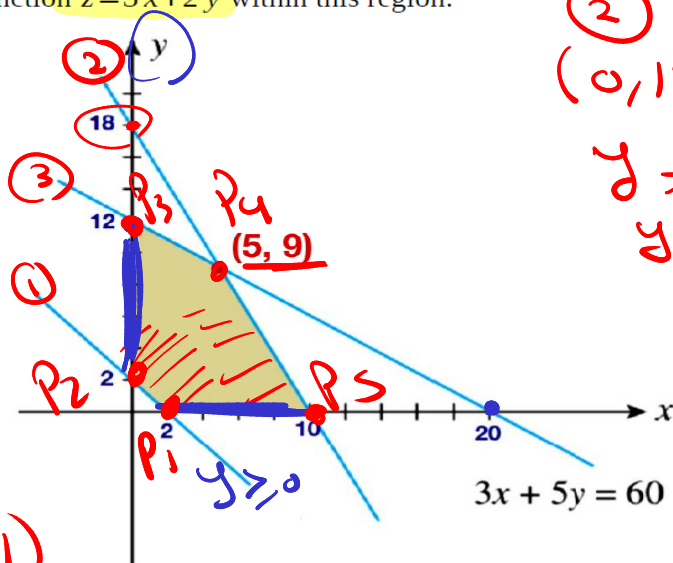
$$x = -3 + 5 + \left[ \frac{8 - 2t - 45}{3} \right]$$

$$x = (-3) + 5 + \left( \frac{8}{3} \right) - \frac{2t}{3} - \left( \frac{45}{3} \right)$$

$$x = -\frac{2t}{3} - \frac{5}{3} - \frac{1}{3}$$

**Question 5** [10 marks]

Write a system of linear inequalities in two variables that describes the shaded region and find the optimal values of the objective function  $z = 3x + 2y$  within this region.



①  
(0,2) (2,0)

$$y = mx + b$$

$$y = mx + 2$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{0 - 2}{2 - 0} = -1$$

$$y = -x + 2$$

$$y + x = 2 \Rightarrow$$

region

$$x + y \geq 2$$

②  
(0,18) (10,0)

$$y = mx + b$$

$$y = mx + 18$$

$$m = \frac{-18}{10}$$

$$y = -\frac{18}{10}x + 18$$

$$10y = -18x + 180$$

$$18x + 10y = 180$$

region

$$18x + 10y \leq 180$$

③

(0,12) (20,0)

$$y = mx + \cancel{b}_{12}$$

$$y = mx + 12$$

$$m = \frac{0 - 12}{20 - 0} = -\frac{12}{20}$$

②0x  $y = -\frac{12}{20}x + 12$

$$20y = -12x + 240$$

$$12x + 20y = 240$$

$$12x + 20y \leq 240$$

$$x + y \geq 2 \quad \text{--- ①}$$

$$18x + 10y \leq 180 \quad \text{--- ②}$$

$$12x + 20y \leq 240 \quad \text{--- ③}$$

$$x \geq 0$$

$$y \geq 0$$

\* optimal value

$$Z = 3x + 2y$$

$$P_1 = (2, 0)$$

$$Z = 3(2) + 2(0) = 6$$

$$P_2 = (0, 2)$$

$$Z = 3(0) + 2(2) = 4$$

$$P_3 = (0, 12)$$

$$Z = 3(0) + 2(12) = 24$$

$$P_4 = (5, 9)$$

$$Z = 3(5) + 2(9) = 33$$

$$P_5 = (10, 0)$$

$$Z = 3(10) + 2(0) = 30$$

\* optimal value.

\* max. value  $Z = 33$  at  $x = 5, y = 9$

\* min value  $Z = 4$  at  $x = 0, y = 2$

**Question 6 [15 marks]**

Solve the following linear programming problem using geometric method.

Minimize and Maximize the objective function  $p$ .

\*  $p = -x + 3y$

Subject to the following constraints.

$2x + y \geq 4$  — ①

$-x + 2y \geq 4$  — ②

$y \leq 6$  — ③

$x, y \geq 0$

$y = 6$

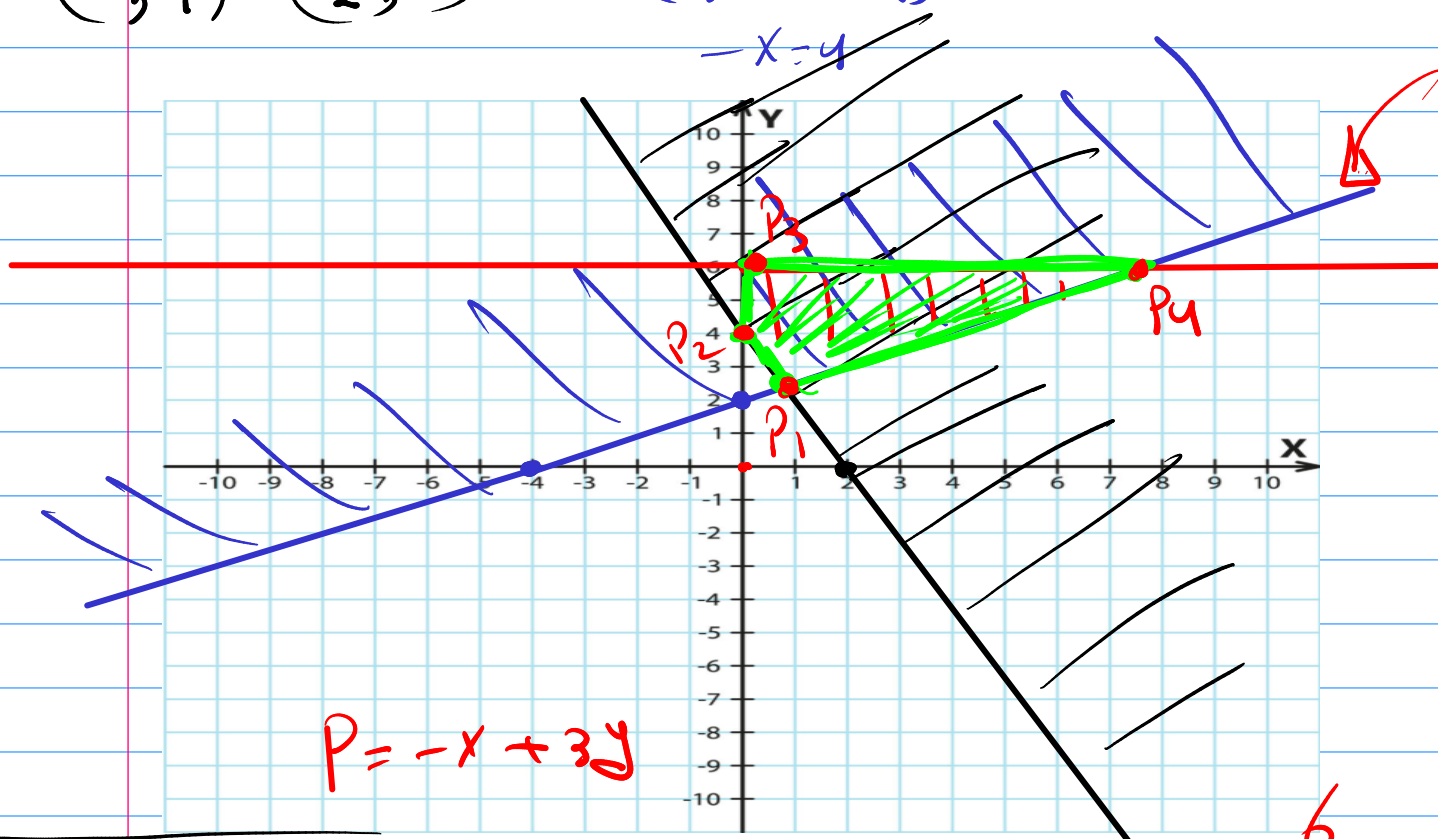
$2x + y = 4$

$(0, 4) (2, 0)$

$-x + 2y = 4$

$(0, 2) (-4, 0)$

$-x = 4$



$P = -x + 3y$

$P_1 = (4/5, 12/5) \rightarrow P = -4/5 + 36/5 = 32/5$

$P_2 = (0, 4) \rightarrow P = 12$

$P_3 (0, 6) \rightarrow P = 18$

$P_4 (8, 6) \rightarrow P = -8 + 18 = 10$

max  $P = 18$  at  $x = 0, y = 6$

min  $P = 32/5$  at  $x = 4/5, y = 12/5$

$-x + 2y = 4$

$-x + 12 = 4$

$-x = 4 - 12$

$x = 8$

$x = 8$

$$\begin{aligned}
 (2*) \quad & 2x + y = 4 \\
 & -x + 2y = 4 \\
 & \hline
 & 2x + 4y = 8 \\
 & 5y = 12 \quad \boxed{y = \frac{12}{5}} \\
 & 2x + \frac{12}{5} = 4 \\
 & 2x = 4 - \frac{12}{5} = \frac{20 - 12}{5} = \frac{8}{5}
 \end{aligned}$$

**Question 7 [15 marks]**

In a laboratory experiment, mice are fed 5 packets of food containing a total 80 units of vitamins E. There are three different brands of food packets that can be used. A packet of brand A contains 5 units of vitamin E, a packet of brand B contains 10 units of vitamin E, and a packet of brand C contains 20 units of vitamin E. How many packets of each brand should be mixed and fed to the mice?

|           | A   | B  | C  |            |
|-----------|-----|----|----|------------|
| Vitamin E | (5) | 10 | 20 | = 80 units |

$x_1 \rightarrow$  number of packets of brand A

$x_2 \Rightarrow$  number of packets of brand B

$x_3 \rightarrow$  number of packets of brand C

$$① \quad x_1 + x_2 + x_3 = 5$$

$$5x_1 + 10x_2 + 20x_3 = 80 \quad \text{--- (2)}$$

$$\begin{aligned}
 & \boxed{x_1 \geq 0} \quad \boxed{x_2 \geq 0} \quad \boxed{x_3 \geq 0}
 \end{aligned}$$

$$\begin{aligned}
 & \cancel{x_1} + 2x_2 + 4x_3 = 16 \\
 & \cancel{x_1} - x_2 - x_3 = -5
 \end{aligned}
 \quad \left. \vphantom{\begin{aligned} \cancel{x_1} + 2x_2 + 4x_3 = 16 \\ \cancel{x_1} - x_2 - x_3 = -5 \end{aligned}} \right\} \text{eq. 1}$$

$$x_2 + 3x_3 = 11$$

$$\boxed{x_2 = 11 - 3x_3}$$

$$11 - 3x_3 \geq 0$$

$$11 \geq 3x_3$$

$$1 \leq x_3 \leq \frac{11}{3}$$

$$1 \leq x_3 \leq 3 \frac{2}{3}$$

$$x_2 = 11 - 3x_3$$

$$x_3 = 1$$

$$x_2 = 8$$

X

$$x_3 = 2$$

$$x_2 = 5$$

X

$$\begin{array}{l} x_3 = 3 \\ x_2 = 2 \\ x_1 = 0 \end{array} \}$$

$$x_1 + x_2 + x_3 = 5$$