

4

Question 1 [15 marks]

Solve and graph on real number line the following compound inequality.

$$\frac{-3}{2} \leq \frac{1}{4} \left(\frac{1}{2}x - 1 \right) + \frac{3}{4} < \frac{3}{2}$$

$$\cancel{\frac{2}{4}} \times \frac{-3}{\cancel{2}} \leq \cancel{\frac{1}{4}} \left(\frac{1}{2}x - 1 \right) + \cancel{\frac{3}{4}} < \cancel{\frac{2}{4}} \times \frac{3}{\cancel{2}}$$

$$-6 \leq \frac{1}{2}x - 1 + 3 < 6$$

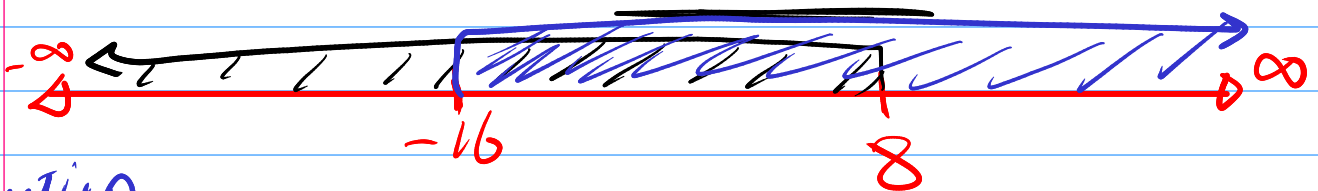
$$-6 \leq \frac{1}{2}x - 1 + 3 < 6$$

$$-6 \leq \frac{1}{2}x + 2 < 6$$

$$-6 - 2 \leq \frac{1}{2}x < 6 - 2$$

$$\cancel{2} \times -8 \leq \cancel{\frac{1}{2}} x < 4 \times 2$$

$$-16 \leq x < 8$$

Solution

$$x \in [-16, 8)$$

Question 2 [10 marks]

Ali invested 12,000 KD in building a restaurant in Kuwait city. The preparation of each meal costs him 3 KD (which includes the ingredients, the labor, the packaging and the delivery).

(i) How many meals should Ali sell in order to recover the initial amount he invested in the building if he sells each meal for 5 KD.

(ii) If he sells 100 meals per day, how many days needed to recover the initial amount he invested in the building.

$$\text{Initial cost} = 12000 \text{ KD}$$

$$\text{Profit of one meal} = 5 - 3 = \boxed{2 \text{ KD}}$$

$$\text{Number of meals} = \frac{12000}{2} = 6000 \text{ meals}$$

$$\Rightarrow 12000 \text{ KD} \Rightarrow 6000 \text{ meals}$$

$$\text{Number of days} = \frac{6000}{100} = \boxed{60} \text{ day}$$

Question 3 [15 marks]

Find an equation of the line passing through the point $(1, 2)$ and intersecting the y -axis at $y=3$.

$$y = mx + b$$

$$y \text{ intercept} = \boxed{y=3}$$

$$\boxed{b=3}$$

$$y = mx + 3$$

at $(1, 2)$

$$2 = m(1) + 3$$

$$2 = m + 3$$

$$m = 2 - 3 = \boxed{-1}$$

$$y = (-1)x + 3$$

solution

$$\boxed{y = -x + 3}$$

Equation of line

*** Question 4 [20 marks]**

Solve by Gauss-Jordan elimination the following system:

$$x + y + z = 1$$

$$2x + y - 3z = 4$$

$$5x + 4y = 7$$

$$[A|B] \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \text{ التوافق }$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 2 & 1 & -3 & 1 & 4 \\ 5 & 4 & 0 & 1 & 7 \end{bmatrix} \xrightarrow{\substack{-2R_1 + R_2 \\ -5R_1 + R_3}}$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & -1 & -5 & -1 & 2 \\ 0 & -1 & -5 & -4 & 2 \end{bmatrix} \quad \begin{aligned} -2(1) + 2 &= 0 \\ -2(1) + 1 &= -2 + 1 = -1 \\ -2 + -3 &= -5 \\ -2 + 4 &= 2 \\ -5(1) + 5 &= 0 \\ -5 + 4 &= -1 \\ -5 & \end{aligned}$$

$$-R_2 + R_3$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & -1 & -5 & -1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

infinity many solutions

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$$R_2 \div -1$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 5 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

x y z

$$-1R_2 + R_1$$

$$z = t$$

$$\begin{bmatrix} 1 & 0 & -4 & 0 & -1 \\ 0 & 1 & 5 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$y + 5z = 2 \Rightarrow y + 5t = 2$$

$$y = 2 - 5t$$

$$X - 4Z = -1 \Rightarrow X - 4t = -1$$

$$\boxed{X = 4t - 1} *$$

Question 5 [10 marks]

Solve the following system of linear inequalities graphically and indicate whether each solution region is bounded or unbounded. Find the co-ordinate of each corner point.

$$x + 4y \leq 32 \rightarrow$$

$$3x + y \leq 30$$

$$4x + 5y \geq 40$$

$$X + 4y = 32$$

$$(0, 8) \quad (32, 0)$$

$$\begin{array}{r} 4y = 32 \\ \hline y = 8 \end{array} \quad \begin{array}{l} X = 32 \\ \hline \end{array}$$

$$3X + y = 30$$

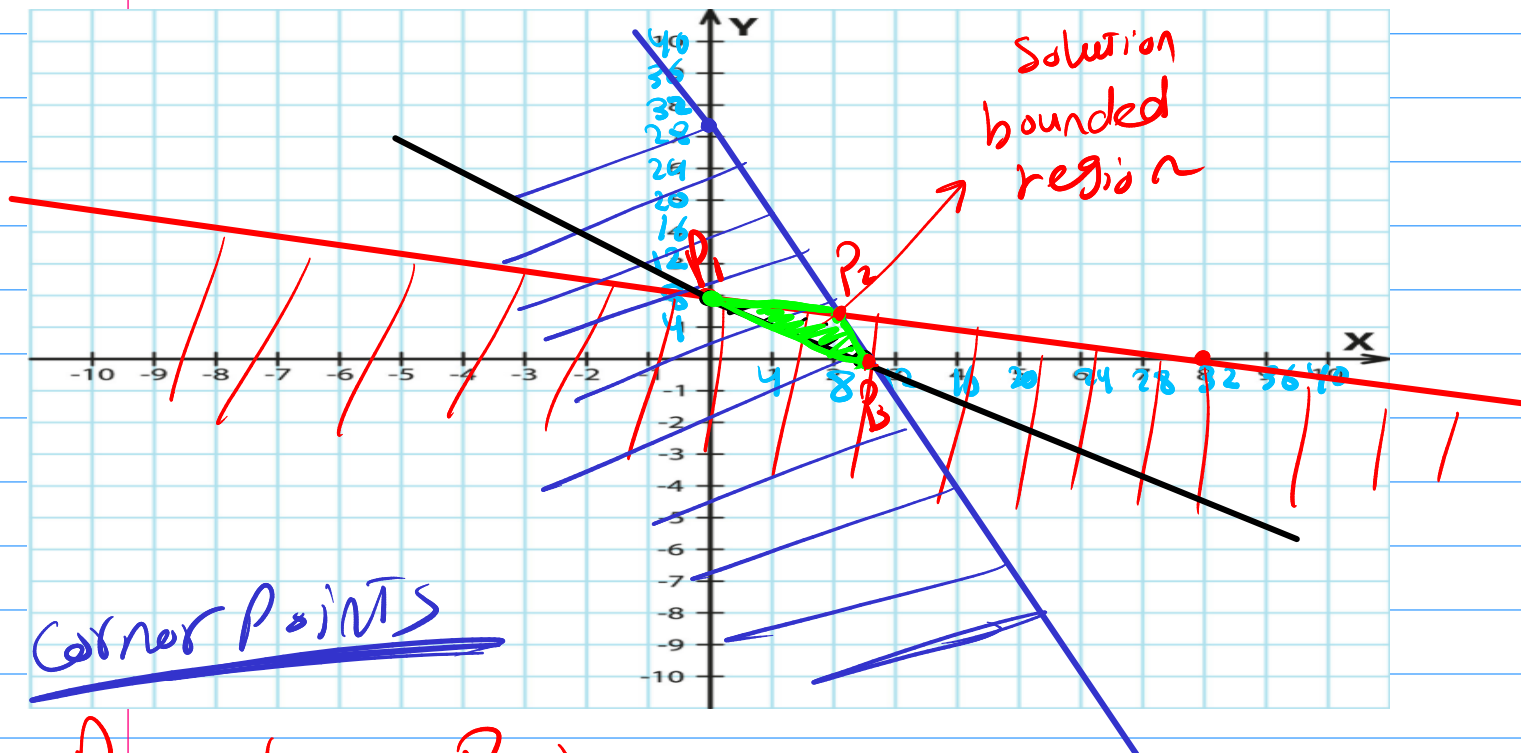
$$(0, 30) \quad (10, 0)$$

$$\begin{array}{r} 3X = 30 \\ \hline X = 10 \end{array}$$

$$4X + 5y = 40$$

$$(0, 8) \quad (10, 0)$$

$$\begin{array}{l} 5y = 40 \mid 4X = 40 \\ \hline y = 8 \mid X = 10 \end{array}$$



$$* P_1 = (0, 8)$$

$$* P_2 = (8, 6)$$

$$* P_3 = (10, 0)$$

$$\begin{array}{r} X + 4y = 32 \\ \hline X + 2y = 32 \\ \hline X = 32 - 2y = \boxed{8} \end{array}$$

$$\begin{array}{r} (-3) * X + 4y = 32 * \\ 3X + y = 30 \\ \hline -3X - 12y = -96 \end{array}$$

$$\begin{array}{r} -11y = -66 \\ \hline y = \boxed{6} \end{array}$$

Question 6 [15 marks]

Minimize and Maximize the objective function: $P = x + 5y$,
subject to

$$3x + 2y \geq 6$$

$$x \geq 0$$

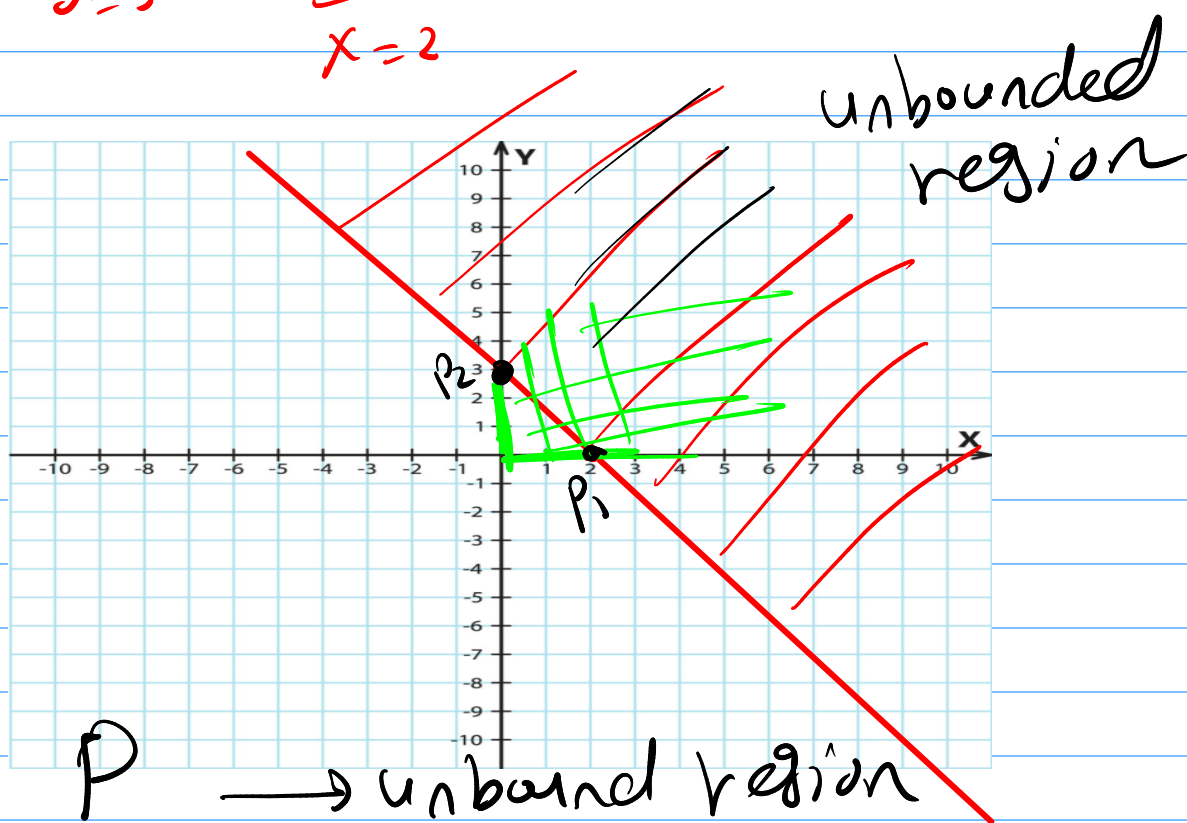
$$y \geq 0$$

$$3x + 2y = 6$$

$$(0, 3) \quad (2, 0)$$

$$2y = 6 \\ y = 3$$

$$\frac{3x}{3} = \frac{6}{3} \\ x = 2$$



max $P \rightarrow$ unbounded region

$$\min \Rightarrow P = x + 5y$$

$$P_2 = (0, 3) \rightarrow 15$$

$$P_1 = (2, 0) \rightarrow 2$$

$$\min P = 2$$

$$x = 2$$

$$y = 0$$

Question 7 [15 marks]

A dietitian is to arrange a special diet composed of three basic foods. The diet is to include exactly 340 units of calcium, 180 units of iron and 220 units of vitamin A. The number of units per ounce of each special ingredient for each of the foods is indicated in the table

	<u>Food A</u> x	<u>Food B</u> y	<u>Food C</u> z	
Calcium	30	10	20	$= 340$
Iron	<u>10</u>	10	20	$= 180$
Vitamin A	10	<u>30</u>	20	$= 220$

How many ounces of each food must be used to meet the diet requirements?

$$\begin{aligned} 30x + 10y + 20z &= 340 \\ 10x + 10y + 20z &= 180 \\ 10x + 30y + 20z &= 220 \end{aligned}$$

$$\begin{aligned} 3x + y + 2z &= 34 \\ x + y + 2z &= 18 \\ x + 3y + 2z &= 22 \end{aligned}$$

augmented matrix

$$\left[\begin{array}{ccc|c} 3 & 1 & 2 & 34 \\ 1 & 1 & 2 & 18 \\ 1 & 3 & 2 & 22 \end{array} \right] \quad R_1 \leftrightarrow R_2$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 18 \\ 3 & 1 & 2 & 34 \\ 1 & 3 & 2 & 22 \end{array} \right] \quad \begin{array}{l} -3R_1 + R_2 \\ -1R_1 + R_3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 18 \\ 0 & -2 & -4 & -20 \\ 0 & 2 & 0 & 4 \end{array} \right] \quad -54 + 34 =$$

$$R_2 \div -2 \quad \left[\begin{array}{ccc|c} 1 & 1 & 2 & 18 \\ 0 & 1 & 2 & 10 \\ 0 & 2 & 0 & 4 \end{array} \right] \quad \begin{array}{l} -R_2 + R_1 \\ -2R_2 + R_3 \end{array}$$

$$-20 \div 4$$

$$\begin{bmatrix} 1 & 0 & 0 & | & 8 \\ 0 & 1 & 2 & | & 10 \\ 0 & 0 & -4 & | & -16 \end{bmatrix}$$

$$R_3 / -4$$

$$\begin{bmatrix} 1 & 0 & 0 & | & 8 \\ 0 & 1 & 2 & | & 10 \\ 0 & 0 & 1 & | & 4 \end{bmatrix}$$

$$(-2)$$

$$-2R_3 + R_2$$

$$\begin{aligned} -2(4) + 10 &= \\ -8 + 10 &= 2 \end{aligned}$$

$$\begin{bmatrix} 1 & 0 & 0 & | & 8 \\ 0 & 1 & 0 & | & 2 \\ 0 & 0 & 1 & | & 4 \end{bmatrix}$$

$$X = 8$$

$$Y = 2$$

$$Z = 4$$

number of ounces of food A = 8

number of ounces of food B = 2

number of ounces of food C = 4