

Question 1 [15 marks]

Solve the following inequality

$$\frac{1}{5}(x+8) < \frac{1}{2}(3x-2) \leq \frac{7}{3}$$

and write the solution in the interval form.

$$5 \times 2 \times 3 = \boxed{30}$$

$$\frac{30}{3} = 10, \quad \frac{30}{5} = 6$$

$$30 \times \frac{1}{2} = \frac{30}{2} = 15$$

$$30 \cdot \frac{1}{5}(x+8) < 30 \cdot \frac{1}{2}(3x-2) \leq 30 \cdot \frac{7}{3}$$
$$6(x+8) < 15(3x-2) \leq 10 \cdot 7$$

$$6x + 48 < 45x - 30 \leq 70$$

$$6x + 48 < 45x - 30 \quad | \quad 45x - 30 \leq 70$$

$$+30+48 < 45x - 6x$$

$$\frac{78}{39} < \frac{39x}{39}$$

$$\boxed{2 < x}$$

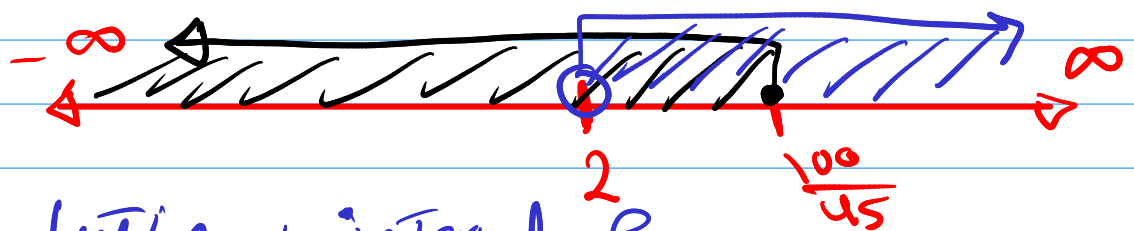
$$45x \leq 70 + 30$$

$$\frac{45x}{45} \leq \frac{100}{45}$$

$$x \leq \frac{100}{45}$$

$$\boxed{x \leq \frac{100}{45}}$$

$$\begin{array}{r} 39x \\ 2 \\ \hline 78 \end{array}$$



Solution in interval form

$$x \in \left(2, \frac{100}{45} \right]$$

Question 2 [10 marks]

Solve the following system

$$4x + 3y = 5 \quad \text{--- ①}$$

$$3x - y = 7 \quad \text{--- ②}$$

(a) using substitution.

(b) using elimination by addition.

اكل بالتعويض

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$$\textcircled{a} \quad 4x + 3y = 5 \quad \text{--- ①}$$

$$3x - y = 7 \Rightarrow$$

$$3x - y = 7$$

$$3x - 7 = y$$

$$y = 3x - 7 \Rightarrow$$

$$4x + 3[3x - 7] = 5$$

$$4x + 9x - 21 = 5$$

$$13x - 21 = 5 \Rightarrow 13x = 21 + 5$$

$$\frac{13x}{13} = \frac{26}{13}$$

$$x = 2$$

$$y = 3x - 7 = 3(2) - 7 = 6 - 7$$

$$y = -1$$

solution

$$x = 2$$

$$y = -1$$

 $\textcircled{b}$

$$4x + 3y = 5$$

$$\textcircled{3} * \quad 3x - y = 7 \Rightarrow$$

$$\begin{aligned} 4x + 3y &= 5 \\ 9x - 3y &= 21 \end{aligned} \quad \left. \begin{array}{l} \text{---} \\ \text{---} \end{array} \right\} \text{المجموع}$$

$$13X = 26$$

$$X = \frac{26}{13} = 2 \quad \boxed{X = 2}$$

$$3(x) - y = 7$$

$$y = 6 - 7$$

$$3(2) - y = 7$$

$$\boxed{y = -1} \neq$$

Question 3 [15 marks]

* Find the slope-intercept form of the equation of the line through the points (2, 5) and (5, 7). Write the resulting equation in the form $Ax + By = C$, where A, B, C are integers and $A > 0$.

$$(2, 5) \quad (5, 7)$$

$$x_1, y_1 \quad x_2, y_2$$

$$y = mx + b$$

$$*(y - y_1) = m(x - x_1)$$

$$m_{\text{slope}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - 5}{5 - 2} = \boxed{\frac{2}{3}}$$

$$(y - y_1) = \frac{2}{3}(x - x_1)$$

$$(y - 5) = \frac{2}{3}(x - 2)$$

$$y - 5 = \frac{2}{3}x - \frac{4}{3}$$

$$y = \frac{2}{3}x - \frac{4}{3} + 5$$

$$\boxed{y = \left(\frac{2}{3}\right)x + \left(\frac{11}{3}\right)}$$

$$\boxed{-\frac{4}{3} + \frac{5}{1} = \frac{-4 + 5}{3 \times 1} = \frac{1}{3}}$$

$$y\text{-intercept} = \frac{11}{3}$$

$$m = \frac{2}{3}$$

$$y = \frac{2}{3}x + \frac{11}{3}$$

$$-\frac{11}{3} = \frac{2}{3}x - y$$

$$\frac{2}{3}x - y = -\frac{11}{3}$$

$$Ax + By = C$$

$$\boxed{A = \frac{2}{3}}$$

$$\boxed{B = -1}$$

$$\boxed{C = -\frac{11}{3}}$$

Question 4 [20 marks]

Solve the following linear system using Gauss-Jordan elimination method:

$$2x_1 + x_2 + 3x_3 = 5$$

$$3x_1 + 2x_2 + 4x_3 = 7$$

$$2x_1 + x_2 + 5x_3 = 10$$

المصفوفة الممتدة Augmented Matrix

$$[A : B] = \left[\begin{array}{ccc|c} 2 & 1 & 3 & 5 \\ 3 & 2 & 4 & 7 \\ 2 & 1 & 5 & 10 \end{array} \right]$$

$$\begin{array}{l} \textcircled{-3} \\ \textcircled{2} \end{array} \left[\begin{array}{ccc|c} 2 & \textcircled{1} & 3 & 5 \\ 3 & \textcircled{2} & 4 & 7 \\ \textcircled{2} & \textcircled{1} & 5 & 10 \end{array} \right]$$

$$\begin{array}{l} -1R_1 + R_3 \\ -3R_1 + 2R_2 \end{array}$$

$$\left[\begin{array}{ccc|c} 2 & 1 & 3 & 5 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 2 & 5 \end{array} \right]$$

$$\begin{array}{l} -3(2) + 2(3) = 0 \\ -3(1) + 2(2) = -3 + 4 \\ -3(3) + 2(4) = -9 + 8 \\ -3(5) + 2(7) = -15 + 14 \end{array}$$

$$R_1/2 \left[\begin{array}{ccc|c} 1 & 1/2 & 3/2 & 5/2 \\ 0 & \textcircled{1} & -1 & -1 \\ 0 & 0 & 2 & 5 \end{array} \right]$$

$$-\frac{1}{2}R_2 + R_1$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 3 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & \textcircled{2} & 5 \end{array} \right]$$

$$\begin{array}{l} -\frac{1}{2}(0) + 1 = 1 \\ -\frac{1}{2}(1) + \frac{1}{2} = 0 \\ -\frac{1}{2}(-1) + \frac{3}{2} \\ \frac{1}{2} + \frac{3}{2} = 2 \end{array}$$

$$R_3/2 \left[\begin{array}{ccc|c} 1 & 0 & \textcircled{2} & 3 \\ 0 & 1 & \textcircled{-1} & -1 \\ 0 & 0 & 1 & 5/2 \end{array} \right]$$

$$\begin{array}{l} R_3 + R_2 \\ -2R_3 + R_1 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & \textcircled{-2} \\ 0 & 1 & 0 & 3/2 \\ 0 & 0 & 1 & 5/2 \end{array} \right]$$

$$\begin{array}{l} 5/2 + -1 = 3/2 \\ -2(5/2) + 3 \\ -5 + 3 = \textcircled{-2} \end{array}$$

$$x_1 = -2$$

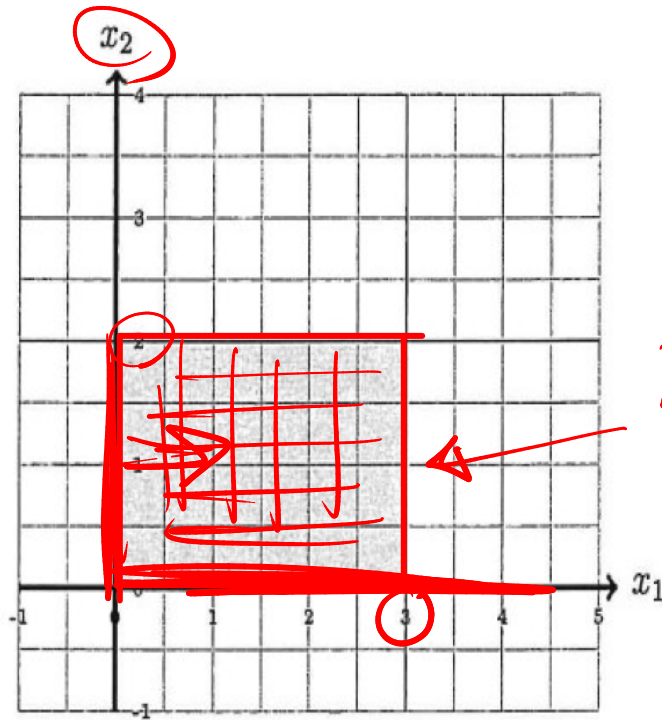
$$x_2 = 3/2$$

$$x_3 = 5/2$$

Question 5 [10 marks]

Write the inequalities defining the following region

$$\begin{aligned}x_2 &\leq 2 \\x_2 &\geq 0 \\x_1 &\geq 0\end{aligned}$$



$$x_1 \leq 3$$

$$\begin{aligned}x_2 &\leq 2 \quad \text{--- ①} \\x_1 &\leq 3 \quad \text{--- ②} \\x_1 &\geq 0 \quad \text{--- ③} \\x_2 &\geq 0 \quad \text{--- ④}\end{aligned}$$

Question 6 [15 marks]

A psychologist uses two types of boxes with mice and rats. The amount of time (in minutes) that each mouse and each rat spends in each box per day is given in the table:

Box	Time: Mice X	Time: Rats Y	Maximum Time Available per Day
Box A	10 min	20 min	400 min
Box B	20 min	10 min	320 min

What is the maximum number of mice and rats that can be used in this experiment? How many mice and how many rats produce this maximum?

* $\max Z = X + Y$ } X : Number of mice
 Y : Number of rats
 $X = ?$ $Y = ?$

$10X + 20Y \leq 400$
 $20X + 10Y \leq 320$
 $X, Y \geq 0$

$10X + 20Y = 400$

$(0, 20) (40, 0)$

$\frac{20Y}{20} = \frac{400}{20} \mid 10X = 400$

$20X + 10Y = 320$

$(0, 32) (16, 0)$

$\frac{10Y}{10} = \frac{320}{10} \mid 20X = 320$

$Y = 32 \mid X = \frac{320}{20} = 16$

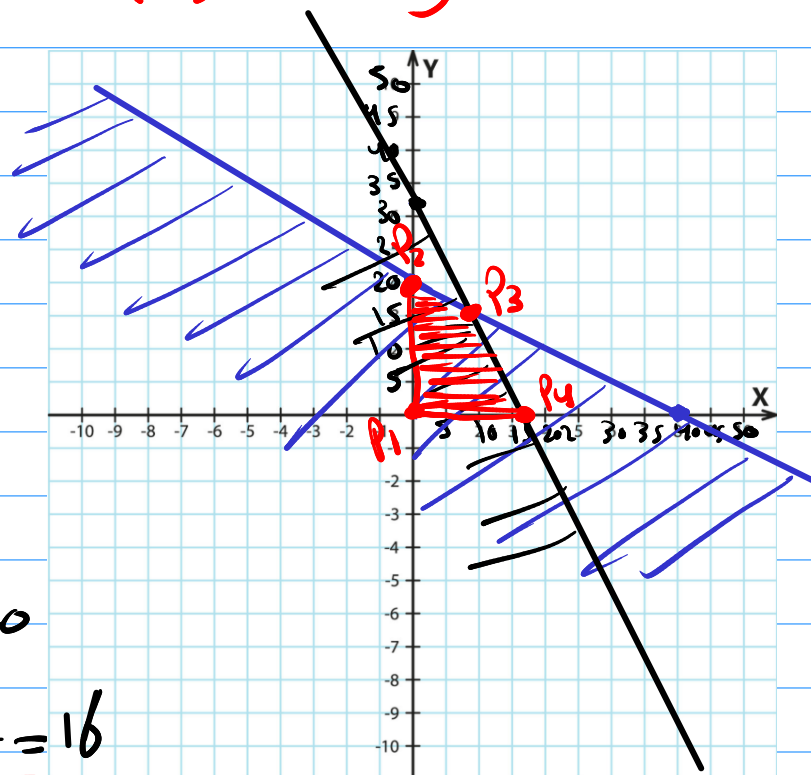
$P_1 (0, 0) \Rightarrow Z = X + Y = 0$

$P_2 (0, 20) \Rightarrow 20$

$P_3 (8, 16) \Rightarrow 24$

$P_4 (16, 0) \Rightarrow 16$

$\max Z = 24$ $X = 8$ $Y = 16$



$-2 \times 10X + 20Y = 400$

$\begin{cases} 20X + 10Y = 320 \\ -20X - 40Y = -800 \end{cases}$

$+30Y = 480$

$Y = \frac{480}{30} = 16$

$10X + 20Y = 400$

$10X + 320 = 400$

Question 7 [15 marks]

A total of 200 coins in 5, 10 and 20 fils denominations must be used to make a payment of 2 KD. Knowing that we have to use at least one coin of each value, find out in how many ways this can be done.

$x \rightarrow$ number of coins 5 fils
 $y \rightarrow$ number of coins 10 fils
 $z \rightarrow$ number of coins 20 fils

$$x + y + z = 200 \quad (1)$$

$$5x + 10y + 20z = 2000 \quad (2)$$

$$x \geq 1, \quad y \geq 1, \quad z \geq 1$$

5 10 20 فليس

$$x + 2y + 4z = 400$$

$$-1 \times x + y + z = 200$$

$$-x - y - z = -200$$

$$x + 2y + 4z = 400$$

$$y + 3z = 200$$

$$y = 200 - 3z$$

$$y \geq 1$$

$$200 - 3z \geq 1$$

$$200 - 1 \geq 3z$$

$$199 \geq 3z$$

$$\frac{3z}{3} \leq \frac{199}{3}$$

$$\boxed{z \leq 66}$$

$$z \geq 1$$

$$66 \geq \textcircled{z} \geq 1$$

لزم z يكون من صحيح

Number of ways 66 way.

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