

Part I: Solve each of the following questions completely(2 points) Evaluate and simplify $\frac{4 - \frac{2}{3}}{\frac{7}{12} - \frac{5}{4}}$.

$$\frac{3 \times 4}{3 \times 1} - \frac{2}{3} = \frac{12}{3} - \frac{2}{3} = \frac{12-2}{3} = \frac{10}{3}$$

$$\frac{7}{12} - \frac{3 \times 5}{3 \times 4} = \frac{7}{12} - \frac{15}{12} = \frac{7-15}{12} = \frac{-8}{12}$$

بغير مضاعف
بسط

$$\left(\frac{\frac{10}{3}}{\frac{-8}{12}} \right) = \frac{10 \times 12}{3 \times -8} = \frac{120}{-24} = -5$$

(10 points) Evaluate $\frac{\frac{1}{3} + \frac{3}{2}}{\frac{7}{10} + \frac{2}{5}}$

$$\frac{1 \times 2}{3 \times 2} + \frac{3 \times 3}{2 \times 3} = \frac{2}{6} + \frac{9}{6} = \frac{2+9}{6} = \frac{11}{6}$$

$$\frac{7}{10} + \frac{2 \times 2}{5 \times 2} = \frac{7}{10} + \frac{4}{10} = \frac{7+4}{10} = \frac{11}{10}$$

$$\left(\frac{\frac{11}{6}}{\frac{11}{10}} \right) = \frac{11 \times 10}{6 \times 11} = \frac{10}{6} = \frac{5}{3}$$

(1.5 points) Simplify $\frac{\frac{2}{5} - \frac{3}{10}}{\frac{1}{2} + \frac{2}{5}}$

$$\frac{\frac{2 \times 2}{5 \times 2} - \frac{3}{10}}{\frac{1 \times 5}{2 \times 5} + \frac{2 \times 2}{5 \times 2}} = \frac{\frac{4}{10} - \frac{3}{10}}{\frac{5}{10} + \frac{4}{10}} = \frac{\frac{4-3}{10}}{\frac{5+4}{10}} = \frac{1}{10} \div \frac{9}{10}$$

$$\left(\frac{\frac{1}{10}}{\frac{9}{10}} \right) = \frac{1 \times 10}{10 \times 9} = \frac{1}{9}$$

(10 points) Simplify the expression $\frac{x^{-2} + \frac{x}{y^3}}{x^{-1} + y^{-1}}$

$$x^{-2} + \frac{x}{y^3} = \frac{y^3}{y^3} \cdot \frac{1}{x^2} + \frac{x \cdot x^2}{y^3 \cdot x^2} = \frac{y^3}{x^2 y^3} + \frac{x^3}{x^2 y^3} = \frac{y^3 + x^3}{x^2 y^3}$$

$$x^{-1} + y^{-1} = \frac{y \cdot 1}{y \cdot x} + \frac{x \cdot 1}{x \cdot y} = \frac{y}{xy} + \frac{x}{xy} = \frac{y+x}{xy}$$

$$\left(\frac{\frac{y^3 + x^3}{x^2 y^3}}{\frac{y+x}{xy}} \right) = \frac{(y^3 + x^3)(xy)}{(x^2 y^3)(y+x)} = \frac{(x+x)(y^2 - xy + x^2)(xy)}{(x^2 y^3)(y+x)} = \frac{y^2 + xy + x^2}{x y^2}$$

نقلب البسط والمقام

نتخلص من الأسس السالبة والكسرية

(2 points) Simplify the expression $\left(\frac{4x^{-1}y^{\frac{1}{3}}}{z}\right)^{-2} \left(\frac{16x^2}{y^{\frac{1}{3}}z^{-5}}\right)$

$$\left(\frac{4x^{-1}y^{\frac{1}{3}}}{z}\right)^{-2} = \left(\frac{z}{4x^{-1}y^{\frac{1}{3}}}\right)^2 = \left(\frac{zx}{4y^{\frac{1}{3}}}\right)^2 = \frac{z^2x^2}{16y^{\frac{2}{3}}}$$

$$\left(\frac{16x^2}{y^{\frac{1}{3}}z^{-5}}\right) = \frac{16x^2z^5}{y^{\frac{1}{3}}}$$

$$\frac{z^2x^2}{16y^{\frac{2}{3}}} \cdot \frac{16x^2z^5}{y^{\frac{1}{3}}} = \frac{z^{2+5}x^{2+2}}{y^{\frac{2}{3}+\frac{1}{3}}} = \frac{z^7x^4}{y^{\frac{3}{3}}} = \frac{z^7x^4}{y}$$

(10 points) Simplify the expression $\left(\frac{3y^3z^{\frac{2}{3}}}{x^{\frac{1}{2}}}\right)^2 \left(\frac{x^{-3}y^6}{27z^4}\right)^{\frac{1}{3}}$

Assume that all letters denote positive numbers.

$$\left(\frac{x^{-3}y^6}{27z^4}\right)^{\frac{1}{3}} = \left(\frac{y^6}{27x^3z^4}\right)^{\frac{1}{3}} = \frac{y^{\frac{6}{3}}}{3x^{\frac{3}{3}}z^{\frac{4}{3}}} = \frac{y^2}{3xz^{\frac{4}{3}}}$$

$$\left(\frac{3y^3z^{\frac{2}{3}}}{x^{\frac{1}{2}}}\right)^2 = \frac{(3)^2 y^{3 \cdot 2} z^{\frac{2}{3} \cdot 2}}{x^{\frac{1}{2} \cdot 2}} = \frac{9y^6z^{\frac{4}{3}}}{x}$$

$$\frac{9y^6z^{\frac{4}{3}}}{x} \cdot \frac{y^2}{3xz^{\frac{4}{3}}} = \frac{y^{6+2}}{x^{1+1}} = \frac{y^8}{x^2}$$

(10 points) Simplify the expression $\left(\frac{3x^2y^{-1}}{2z^{-3}}\right)^{-3} \left(\frac{x^{\frac{-1}{2}}y^2}{z}\right)^2$,

where x , y and z are positive real numbers.

$$\left(\frac{3x^2y^{-1}}{2z^{-3}}\right)^{-3} = \left(\frac{2z^{-3}}{3x^2y^{-1}}\right)^3 = \left(\frac{2y}{3z^3x^2}\right)^3 = \frac{8y^3}{27z^9x^6}$$

$$\left(\frac{x^{\frac{-1}{2}}y^2}{z}\right)^2 = \left(\frac{y^2}{x^{\frac{1}{2}}z}\right)^2 = \frac{y^4}{x^{\frac{1}{2} \cdot 2} z^2} = \frac{y^4}{x z^2}$$

$$\frac{8y^3}{27z^9x^6} \cdot \frac{y^4}{xz^2} = \frac{8y^{3+4}}{27z^{9+2}x^{6+1}} = \frac{8y^7}{27z^{11}x^7}$$

(2 points) Simplify the expression $\left(\frac{2x^{\frac{1}{2}}}{y^{\frac{4}{3}}}\right)^3 \left(\frac{y^2}{x^{\frac{-1}{2}}}\right)$.

$$\left(\frac{2x^{\frac{1}{2}}}{y^{\frac{4}{3}}}\right)^3 = \frac{(2)^3 x^{\frac{3}{2}}}{y^{\frac{12}{3}}} = \frac{8x^{\frac{3}{2}}}{y^4}$$

$$\left(\frac{y^2}{x^{\frac{-1}{2}}}\right) = y^2 \cdot x^{\frac{1}{2}}$$

$$\frac{8x^{\frac{3}{2}}}{y^4} \cdot y^2 \cdot x^{\frac{1}{2}} = \frac{8x^{\frac{3}{2} + \frac{1}{2}}}{y^{4-2}} = \frac{8x^2}{y^2}$$

نحول : ← ضرب
ونقلب العكس الثاني

(2 points) Perform the division and simplify.

$$\frac{x - 5}{2x^2 - 5x - 12} \div \frac{x^2 - 9x + 20}{2x + 3}$$

$$= \frac{(x - 5)}{2x^2 - 5x - 12} \cdot \frac{2x + 3}{x^2 - 9x + 20}$$

$$= \frac{\cancel{x - 5}}{(2x + 3)(x - 4)} \cdot \frac{\cancel{2x + 3}}{(x - 5)(x - 4)}$$

$$= \frac{1}{(x - 4)^2}$$

(3 points) Simplify $\frac{5x + 10}{x^2 - 2x - 3} \div \frac{x^2 - 4}{x + 1}$

$$\frac{5x + 10}{x^2 - 2x - 3} \cdot \frac{x + 1}{x^2 - 4}$$

$$= \frac{5(\cancel{x + 2})}{(x - 3)(\cancel{x + 1})} \cdot \frac{\cancel{(x + 1)}}{(\cancel{x + 2})(x - 2)}$$

$$= \frac{5}{(x - 3)(x - 2)}$$

(2 points) Simplify the expression $\frac{x^2+7x+12}{2x+6} \div \frac{x^2-16}{x^2+1}$.

$$\begin{aligned} & \frac{x^2+7x+12}{2x+6} \cdot \frac{x^2+1}{x^2-16} \\ & = \frac{(x+3)(x+4)}{2(x+3)} \cdot \frac{x^2+1}{(x+4)(x-4)} \\ & = \frac{x^2+1}{2(x-4)} \end{aligned}$$

فرق بين مربعين

أقواس

(7.5 + 7.5 = 15 points) Factor each of the following completely.

a) $8x^4y - 27xy^4$. أقل أس تكرر

$$\begin{aligned} & = xy(8x^3 - 27y^3) = xy((2x)^3 - (3y)^3) \\ & = xy[(2x-3y)(4x^2+6xy+9y^2)] \end{aligned}$$

عكس الإشارة

b) $4x^3 + 12x^2 - x - 3$

$$\begin{aligned} & = 4x^2(x+3) - (x+3) \\ & = (x+3)(4x^2 - 1) \\ & = (x+3)((2x)^2 - 1) \\ & = (x+3)(2x+1)(2x-1) \end{aligned}$$

فرق بين مربعين

(3 points) Factor completely.

a. $2x^4 + 16x$

b. $9x^3 - 18x^2 + x - 2$

$$\begin{aligned} \text{a. } & 2x(x^3 + 8) \text{ مجموع مكعبين} \\ & = 2x(x^3 + 2^3) \\ & = 2x(x + 2)(x^2 - 2x + 4) \end{aligned}$$

$$\begin{aligned} \text{b. } & 9x^2(x-2) + (x-2) \\ & = (x-2)(9x^2 + 1) \end{aligned}$$

a. Factor $t^3 + 3t^2 - 4t - 12$ completely.

$$\begin{aligned} & = t^2(t+3) - [4(t+3)] \\ & = (t+3)(t^2 - 4) \text{ فرق بين مربعين} \\ & = (t+3)(t+2)(t-2) \end{aligned}$$

Compute $(7-i)(4+2i)$

$$\begin{aligned} & = 28 + 14i - 4i - 2i^2 \\ & = 28 + 10i - 2i^2 \end{aligned}$$

(3 points) Factor each of the following completely.

a. $(x^3 - 27)$ فرق بين مكعبين
 $= (x^3 - 3^3)$
 $= (x - 3)(x^2 + 3x + 9)$

b. $(x^3 - 2x^2 + 4x - 8)$
 $= x^2(x - 2) + 4(x - 2)$
 $= (x - 2)(x^2 + 4)$

(3 points) Factor the expression completely.

a. $x^{\frac{3}{2}} - 6x^{\frac{1}{2}} - 16x^{-\frac{1}{2}}$ الأس الأقل ← سلب
 $= x^{-\frac{1}{2}}(x^{\frac{3}{2} - (-\frac{1}{2})} - 6x^{\frac{1}{2} - (-\frac{1}{2})} - 16)$
 $= x^{-\frac{1}{2}}(x^{\frac{3+1}{2}} - 6x^{\frac{1+1}{2}} - 16)$
 $= x^{-\frac{1}{2}}(x^{\frac{4}{2}} - 6x^{\frac{2}{2}} - 16)$
 $= x^{-\frac{1}{2}}(x^2 - 6x - 16)$
 $= x^{-\frac{1}{2}}[(x - 8)(x + 2)] = \frac{1}{\sqrt{x}}(x - 8)(x + 2)$

المشاركين بينهم
(أشاروا)

$$\begin{aligned} \text{b. } & \underline{2x^3 - x^2 - 18x + 9} \\ & = x^2(2x - 1) - 9(2x - 1) \\ & = (2x - 1)(x^2 - 9) \\ & = (2x - 1)(x - 3)(x + 3) \end{aligned}$$

Factor the expression completely

$$\text{a. } \underline{x^3 - 5x^2 - 4x + 20}$$

$$\text{b. } \left(a + \frac{4}{b^2}\right)^2 - \left(a - \frac{4}{b^2}\right)^2$$

$$\begin{aligned} \text{a. } & \underline{x^2(x - 5) - 4(x - 5)} \\ & = (x - 5)(x^2 - 4) \quad \text{فرق بين مربعين} \\ & = (x - 5)(x^2 - 2^2) \\ & = (x - 5)(x - 2)(x + 2) \end{aligned}$$

$$\begin{aligned} \text{b. } & \left(a + \frac{4}{b^2}\right)^2 - \left(a - \frac{4}{b^2}\right)^2 \quad \text{فرق بين مربعين} \\ & = \left[\left(a + \frac{4}{b^2}\right) + \left(a - \frac{4}{b^2}\right)\right] \left[\left(a + \frac{4}{b^2}\right) - \left(a - \frac{4}{b^2}\right)\right] \\ & = 2a \left[\cancel{a} + \frac{4}{b^2} - \cancel{a} + \frac{4}{b^2}\right] \\ & = 2a \left(\frac{8}{b^2}\right) = \frac{16a}{b^2} \end{aligned}$$

(10 points) Find all solutions of the equation $x^2 - 4x - 1 = 0$.

المميز

$$ax^2 + bx + c = 0$$

$$a = 1, b = -4, c = -1$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-1)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{16 + 4}}{2} = \frac{4 \pm \sqrt{20}}{2} = \frac{4 \pm \sqrt{4 \cdot 5}}{2}$$

$$= \frac{4 \pm \sqrt{4} \sqrt{5}}{2} = \frac{4 \pm 2\sqrt{5}}{2} = 2 \pm \sqrt{5}$$

$$S = \{2 - \sqrt{5}, 2 + \sqrt{5}\}$$

(15 points) Find the solution set of the equation $2x^2 - 5x + 1 = 0$

$$ax^2 + bx + c = 0$$

$$a = 2, b = -5, c = 1$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(1)}}{2(2)}$$

$$= \frac{5 \pm \sqrt{25 - 8}}{4} = \frac{5 \pm \sqrt{17}}{4}$$

$$S = \left\{ \frac{5 + \sqrt{17}}{4}, \frac{5 - \sqrt{17}}{4} \right\}$$

Find the solution set of the equation $x^2 + 2x - 4 = 0$.

$$ax^2 + bx + c = 0$$

$$a = 1, b = 2, c = -4$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(-4)}}{2(1)}$$

$$= \frac{-2 \pm \sqrt{4 + 16}}{2} = \frac{-2 \pm \sqrt{20}}{2}$$

$$= \frac{-2 \pm \sqrt{4 \cdot 5}}{2} = \frac{-2 \pm \sqrt{4} \sqrt{5}}{2} = \frac{-2 \pm 2\sqrt{5}}{2}$$

$$= -1 \pm \sqrt{5}, \text{ } \{ -1 + \sqrt{5}, -1 - \sqrt{5} \}$$

(3 points) Solve the following equation.

$$x^4 - 5x^2 + 4 = 0$$

$$\text{let } w^2 = x^4$$

$$w \neq x^2$$

أخلى المعادلة من الدرجة الثانية
أفرض $w^2 = x^4$

$$w^2 - 5w + 4 = 0$$

$$(w - 4)(w - 1) = 0$$

$$w - 4 = 0$$

$$w = 4$$

$$\therefore \sqrt{x^2} = \sqrt{4}$$

$$x = \pm 2$$

or

$$w - 1 = 0$$

$$w = 1$$

$$\sqrt{x^2} = \sqrt{1}$$

$$x = \pm 1$$

$$\{ -2, -1, 1, 2 \}$$

(2.5 points) Find the solution of the equation:

$$x^2 + 6x - 3 = 0$$

$$ax^2 + bx + c = 0$$

$$a = 1, b = 6, c = -3$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(6) \pm \sqrt{(6)^2 - 4(1)(-3)}}{2(1)}$$

$$= \frac{-6 \pm \sqrt{36 + 12}}{2} = \frac{-6 \pm \sqrt{48}}{2}$$

$$= \frac{-6 \pm \sqrt{3 \cdot 16}}{2} = \frac{-6 \pm 4\sqrt{3}}{2} = -3 \pm 2\sqrt{3}$$

$$S = \{-3 - 2\sqrt{3}, -3 + 2\sqrt{3}\}$$

(10 points) Find all solutions of the inequality $|5x + 3| \geq 2$.

$$5x + 3 \leq -2 \quad \text{or} \quad 5x + 3 \geq 2$$

$$5x \leq -2 - 3$$

$$\cancel{5}x \leq \cancel{-5}$$

$$x \leq -1$$

$$(-\infty, -1]$$

$$5x \geq 2 - 3$$

$$\cancel{5}x \geq \cancel{-1}$$

$$x \geq -\frac{1}{5}$$

$$[-\frac{1}{5}, \infty)$$

$$S = (-\infty, -1] \cup [-\frac{1}{5}, \infty)$$

(15 points) Find the solution of the inequality

$$\frac{x-5}{x^2-3x+2} \leq 0.$$

$$\frac{x-5}{x^2-3x+2} \leq 0$$

$$\frac{(x-5)}{(x-2)(x-1)} \leq 0$$

أصفاء المقام { أصفاء البسط
 $x-5=0$ $x-2=0, x-1=0$
 $x=5$ $x=2, x=1$

	$-\infty$		1		2		5		∞
$x-5$	-	-	-	-	-	-	+	+	+
$x-2$	-	-	-	-	+	+	+	+	+
$x-1$	-	-	+	+	+	+	+	+	+
$\frac{x-5}{(x-2)(x-1)}$	-	-	+	+	-	-	+	+	+

المتباينة أقل من الصفر
 نختار فترات سالبة

$$x \in (-\infty, 1) \cup (2, 5]$$

(2 points) Find the solution of the inequality

$$x^3 - 5x^2 - 14x < 0$$

$$= x(x^2 - 5x - 14) < 0$$

$$= x(x-7)(x+2) < 0$$

المتباينة أقل من الصفر
 نبحث عن فترات سالبة

$$x \in (-\infty, -2) \cup (0, 7)$$

Zero $x=0$ { $x-7=0$ { $x+2=0$
 $x=7$ { $x=-2$

	$-\infty$		-2		0		7		∞
x	-	-	-	-	+	+	+	+	+
$x-7$	-	-	-	-	-	-	+	+	+
$x+2$	-	-	+	+	+	+	+	+	+
$x(x-7)(x+2)$	-	-	+	+	-	-	+	+	+

(15 points) Find the solution of the inequality

$$\frac{x-5}{x^2-3x+2} \leq 0.$$

$$\frac{x-5}{x^2-3x+2} \leq 0$$

$$\frac{(x-5)}{(x-2)(x-1)} \leq 0$$

Zero

أصفر البسط $x-5=0$
 $x=5$

أصفر المقام $x-2=0$
 $x=2$

$(x-1)=0$
 $x=1$

	$-\infty$	0	2	5	∞
$x-5$	-	-	-	+	
$x-2$	-	-	+	+	
$x-1$	-	+	+	+	
$\frac{x-5}{(x-2)(x-1)}$	-	+	-	+	

المتباينة أقل من أو تساوي صفر
 تبين من فترات سالبة

$$x \in (-\infty, 1) \cup (2, 5]$$

(2 points) Find the solution of the inequality

$$x^3 - 5x^2 - 14x < 0$$

$$x(x^2 - 5x - 14) < 0$$

$$x(x-7)(x+2) < 0$$

المتباينة أقل من الصفر
 تبين من فترات سالبة

$$x \in (-\infty, -2) \cup (0, 7)$$

Zero $x=0$, $x-7=0$, $x+2=0$
 $x=7$ $x=-2$

	$-\infty$	-2	0	7	∞
x	-	-	+	+	
$x-7$	-	-	-	+	
$x+2$	-	+	+	+	
$x(x-7)(x+2)$	-	+	-	+	

2. (3 points) Find the solution of the inequality $\frac{x}{x-1} \geq 2$ لازم أخيراً منقحة

$$\frac{x}{x-1} - \frac{2}{1} \geq 0$$

$$\frac{x}{x-1} - \frac{2(x-1)}{x-1} \geq 0$$

$$\frac{x - 2x + 2}{x-1} \geq 0$$

$$\frac{-x + 2}{x-1} \geq 0$$

أصل المعادلات

$$\begin{cases} -x + 2 = 0 \\ x - 1 = 0 \end{cases} \Rightarrow \begin{cases} x = 2 \\ x = 1 \end{cases}$$

إشارة x سالبة
بعض سالبة
سار موجب

	$-\infty$	1	2	∞
$-x+2$	+	0	+	-
$x-1$	-	0	+	+
$\frac{-x+2}{x-1}$	-	+	-	-

$$x \in (1, 2]$$

المتباينة أكبر من الصفر
نبحث عن فترات موجبة

(2 points) solve the following inequality.

$$x^3 - 3x^2 - 18x \leq 0$$

$$= x(x^2 - 3x - 18) \leq 0$$

$$= x(x - 6)(x + 3) \leq 0$$

Zero $x = 0$

$$\begin{cases} x - 6 = 0 \\ x + 3 = 0 \end{cases} \Rightarrow \begin{cases} x = 6 \\ x = -3 \end{cases}$$

	$-\infty$	-3	0	6	∞
x	-	-	+	+	
$x-6$	-	-	-	+	
$x+3$	-	+	+	+	
$x(x-6)(x+3)$	-	+	-	+	

المتباينة أقل من الصفر
نبحث عن فترات سالبة

$$x \in (-\infty, -3] \cup [0, 6]$$

أخذهامفوية

(2.5 points) Find the solution of the inequality $\frac{5x-3}{x+2} \leq 1$.

$$\frac{5x-3}{x+2} - \frac{1}{1} \leq 0$$

$$\frac{5x-3}{x+2} - \frac{x+2}{x+2} \leq 0$$

$$\frac{5x-3-x-2}{x+2} \leq 0$$

$$\frac{4x-5}{x+2} \leq 0$$

Zero أمنا، البط

$$4x-5=0$$

$$4x=5$$

$$x=\frac{5}{4}$$

أمنا، المعام

$$x+2=0$$

$$x=-2$$

	$-\infty$	-2	$\frac{5}{4}$	$+\infty$
$4x-5$	-	-	+	+
$x+2$	-	+	+	+
$\frac{4x-5}{x+2}$	+	-	+	+

المبتاينة أقل من الصفر
نبحث عن فترات سالبة

$$x \in (-2, 5/4]$$

(1.5 + 1 + 2.5 points) Given the points A(2, 1) and B(4, -3).

 $x_1 \ y_1$ $x_2 \ y_2$

a. Find the distance between A and B.

b. Find the midpoint of the line segment $\overline{AB}$.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(4-2)^2 + (-3-1)^2} = \sqrt{(2)^2 + (-4)^2} = \sqrt{4+16}$$

$$= \sqrt{20} = 2\sqrt{5}$$

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$= \left(\frac{2+4}{2}, \frac{1+(-3)}{2} \right) = \left(\frac{6}{2}, \frac{-2}{2} \right) = (3, -1)$$

(7.5 + 7.5 = 15 points) Let $A(x_1, y_1)$ and $B(x_2, y_2)$. Find:

(a) The distance between A and B .

(b) The midpoint of the line segment joining A and B

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(5 - 4)^2 + (-3 - 2)^2}$$

$$= \sqrt{(1)^2 + (-5)^2} = \sqrt{1 + 25} = \sqrt{26}$$

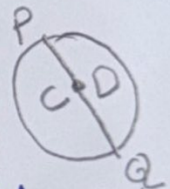
$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left(\frac{4 + 5}{2}, \frac{2 - 3}{2} \right)$$

$$= \left(\frac{9}{2}, -\frac{1}{2} \right)$$

(3 points) Find an equation of the circle that has the points $P(x_1, y_1)$ and $Q(x_2, y_2)$ as endpoints of a diameter.

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$= \left(\frac{-6 + 2}{2}, \frac{2 + 2}{2} \right) = \left(\frac{-4}{2}, \frac{4}{2} \right) = (-2, 2) \Rightarrow \text{Center}$$



عندما يكون
نقطتين

$$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(2 - (-6))^2 + (2 - 2)^2} = \sqrt{(8)^2 + 0} = \sqrt{16} = 4$$

$$r^2 = 16$$

$$(x - h)^2 + (y - k)^2 = r^2$$

$$(x - (-2))^2 + (y - 2)^2 = 16$$

$$(x + 2)^2 + (y - 2)^2 = 16$$

center
أحتاج r^2

(3 points) Find an equation of the circle that has center $(4, -1)$ and passes through $(6, 2)$.

إذا كان فني
center + نقطة

$$\begin{aligned} r^2 &= (x-h)^2 + (y-k)^2 \\ &= (6-4)^2 + (2-(-1))^2 \\ &= (2)^2 + (3)^2 = 4 + 9 = 13 \end{aligned}$$

معادلة
الدائرة

$$\begin{aligned} (x-h)^2 + (y-k)^2 &= r^2 \\ (x-4)^2 + (y+1)^2 &= 13 \end{aligned}$$

(10 points) Find an equation of the circle that has center $(7, -2)$ and passing through $(4, 1)$.

$$\begin{aligned} r^2 &= (x-h)^2 + (y-k)^2 \\ &= (4-7)^2 + (1-(-2))^2 \\ &= (-3)^2 + (3)^2 = 9 + 9 = 18 \end{aligned}$$

معادلة
الدائرة

$$\begin{aligned} (x-h)^2 + (y-k)^2 &= r^2 \\ (x-7)^2 + (y+2)^2 &= 18 \end{aligned}$$

slope ← m
[تحتاج] نقطة

(2 points) Find an equation of the line passing through the points $P(1, -3)$ and $Q(-2, 6)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - (-3)}{-2 - 1} = \frac{6 + 3}{-3} = \frac{9}{-3} = -3$$

معادلة
Line

$$y - y_1 = m(x - x_1)$$

$$y + 3 = -3(x - 1)$$

$$y + 3 = -3x + 3$$

$$y = -3x + 3 - 3$$

$$y = -3x$$

(10 points) Find an equation of the line passing through the points $P(1, 3)$ and $Q(-1, -4)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4 - 3}{-1 - 1} = \frac{-7}{-2} = \frac{7}{2}$$

معادلة
Line

$$y - y_1 = m(x - x_1)$$

$$(y - 3) = \frac{7}{2}(x - 1)$$

$$y - 3 = \frac{7}{2}x - \frac{7}{2}$$

$$y = \frac{7}{2}x - \frac{7}{2} + \frac{2 \times 3}{2}$$

$$y = \frac{7}{2}x - \frac{7}{2} + \frac{6}{2}$$

$$y = \frac{7}{2}x - \frac{1}{2}$$

(2 points) Find an equation of the line through the point $P(3, 5)$ that is parallel to the line $3x + 2y - 1 = 0$.

$$Ax + By + C = 0$$

$$m_{L_2} = -\frac{A}{B} = -\frac{3}{2}$$

$$\therefore L_1 \parallel L_2$$

$$\therefore m_{L_2} = m_{L_1} \Rightarrow m_{L_1} = -\frac{3}{2}$$

معادلة
Line

$$y - y_1 = m(x - x_1)$$

$$y - 5 = -\frac{3}{2}(x - 3)$$

$$y - 5 = -\frac{3}{2}x + \frac{9}{2}$$

$$y = -\frac{3}{2}x + \frac{9}{2} + \frac{2 \times 5}{2}$$

$$y = -\frac{3}{2}x + \frac{9}{2} + \frac{10}{2}$$

$$y = -\frac{3}{2}x + \frac{19}{2}$$

نقطة m, L_1

(10 points) Find an equation of the line passing through the point $P(-5, 2)$ and parallel to the line whose equation is $4x - 3y - 1 = 0$
 $Ax + By + C = 0$

$$m_{L_2} = \frac{-A}{B}$$

$$= \frac{-4}{-3} = \frac{4}{3}$$

$$\therefore L_2 \parallel L_1$$

$$\therefore m_{L_2} = m_{L_1} \Rightarrow m_{L_1} = \frac{4}{3}$$

معادلة
Line

$$y - y_1 = m(x - x_1)$$

$$(y - 2) = \frac{4}{3}(x + 5)$$

$$y - 2 = \frac{4}{3}x + \frac{20}{3}$$

$$y = \frac{4}{3}x + \frac{20}{3} + \frac{3 \times 2}{3 \times 1}$$

$$y = \frac{4}{3}x + \frac{20}{3} + \frac{6}{3}$$

$$y = \frac{4}{3}x + \frac{26}{3}$$