

EXAMPLE 3**Nutrition**

A patient on a brown rice and skim milk diet is required to have at least 800 calories and at least 32 grams of protein per day. Each serving of brown rice contains 200 calories and 5 grams of protein. Each serving of skim milk contains 80 calories and 8 grams of protein. How many servings of each food should be eaten per day to meet the minimum daily requirements?

number

brown rice

milk

total

Calories
protein

200

80

 ≥ 800

5

8

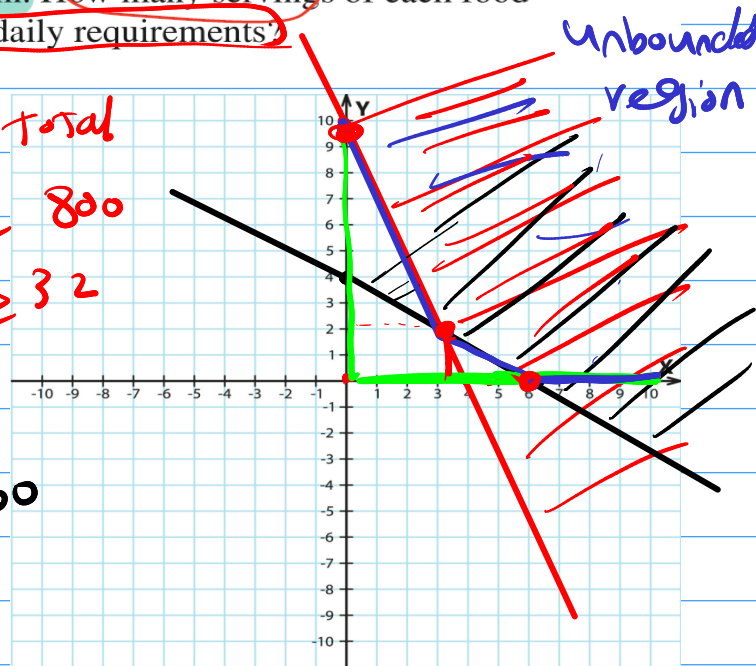
 ≥ 32

$$\min Z = x + y$$

$$200x + 80y \geq 800$$

$$5x + 8y \geq 32$$

$$x \geq 0 \quad y \geq 0$$



$$200x + 80y = 800$$

$$(0, 10) \quad (4, 0)$$

$$\frac{80y = 800}{80} \quad \frac{800}{80}$$

$$y = 10$$

$$\frac{200x = 800}{200} \quad \frac{800}{200}$$

$$x = 4$$

$$5x + 8y = 32$$

$$(0, 4) \quad (6.4, 0)$$

$$8y = 32$$

$$y = 4$$

$$5x = 32$$

$$x = \frac{32}{5} = 6.4$$

$$P_1 = (0, 10)$$

$$P_2 = (3.2, 2)$$

$$P_3 = (6, 0)$$

$$Z = x + y = 10$$

$$Z = 5.2$$

$$Z = 6$$

min value

$$x = 3.2$$

$$y = 2$$

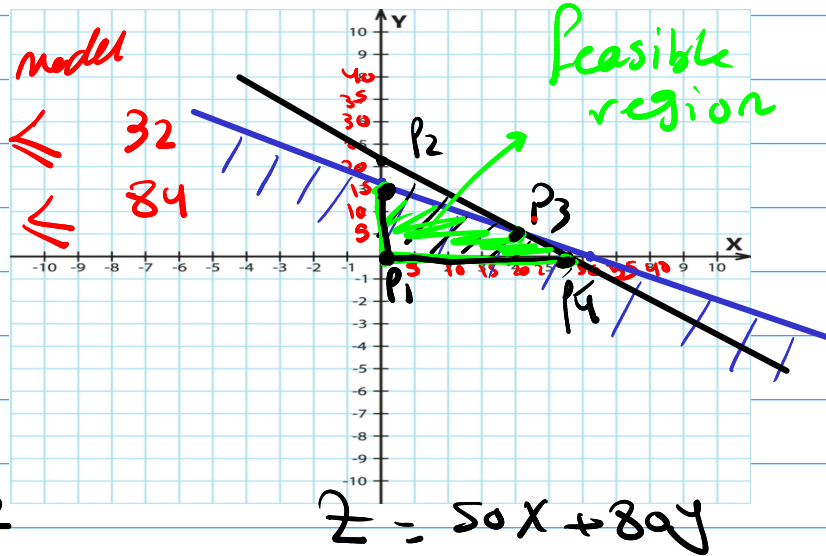
$$x \leq 3$$

$$y \leq 2$$

EXAMPLE 1**Production Scheduling**

A manufacturer of lightweight mountain tents makes a standard model and an expedition model. Each standard tent requires 1 labor-hour from the cutting department and 3 labor-hours from the assembly department. Each expedition tent requires 2 labor-hours from the cutting department and 4 labor-hours from the assembly department. The maximum labor-hours available per day in the cutting and assembly departments are 32 and 84, respectively. If the company makes a profit of \$50 on each standard tent and \$80 on each expedition tent, how many tents of each type should be manufactured each day to maximize the total daily profit (assuming that all tents can be sold)?

Number	X	Y
	Standard model	expedition model
cutting	1	2
assembly	3	4



$$\max Z = 50X + 80Y$$

$$X + 2Y \leq 32$$

$$3X + 4Y \leq 84$$

$$X \geq 0$$

$$Y \geq 0$$

$$X + 2Y = 32$$

$$(0, 16) \quad (32, 0)$$

$$3X + 4Y = 84$$

$$(0, 21) \quad (28, 0)$$

$$\frac{4Y}{4} = \frac{84}{4} \quad | \quad 3X = 84$$

$$Y = 21$$

$$X = 28$$

$$\max Z = 1480$$

$$X = 20 \quad Y = 6$$

$$P_1 = (0, 0) \rightarrow Z = 0$$

$$P_2 = (0, 16) \rightarrow Z = 1280$$

$$P_3 = (20, 6) \rightarrow Z = 1480$$

$$P_4 = (28, 0) \rightarrow Z = 1400$$

$$X + 2Y = 32$$

$$3X + 4Y = 84$$

$$-3X - 6Y = -96$$

$$2Y = -12$$

$$Y = -6$$

$$Y = 6$$

$$X + 2Y = 32$$

$$X = 32 - 2(6)$$

$$X = 32 - 12$$

$$X = 20$$

EXAMPLE 3**Medication**

A hospital patient is required to have at least 84 units of drug A and 120 units of drug B each day (assume that an overdose of either drug is harmless). Each gram of substance M contains 10 units of drug A and 8 units of drug B, and each gram of substance N contains 2 units of drug A and 4 units of drug B. Now, suppose that both M and N contain an undesirable drug D: 3 units per gram in M and 1 unit per gram in N. How many grams of each of substances M and N should be mixed to meet the minimum daily requirements and simultaneously minimize the intake of drug D? How many units of the undesirable drug D will be in this mixture?

	x M	y N
drug A	10	2
drug B	8	4
drug D	3	1

$$\min Z = 3x + y$$

$$10x + 2y \geq 84$$

$$8x + 4y \geq 120$$

$$x \geq 0 \quad y \geq 0$$

$$10x + 2y = 84$$

$$(0, 42) \quad (8.4, 0)$$

$$8x + 4y = 120$$

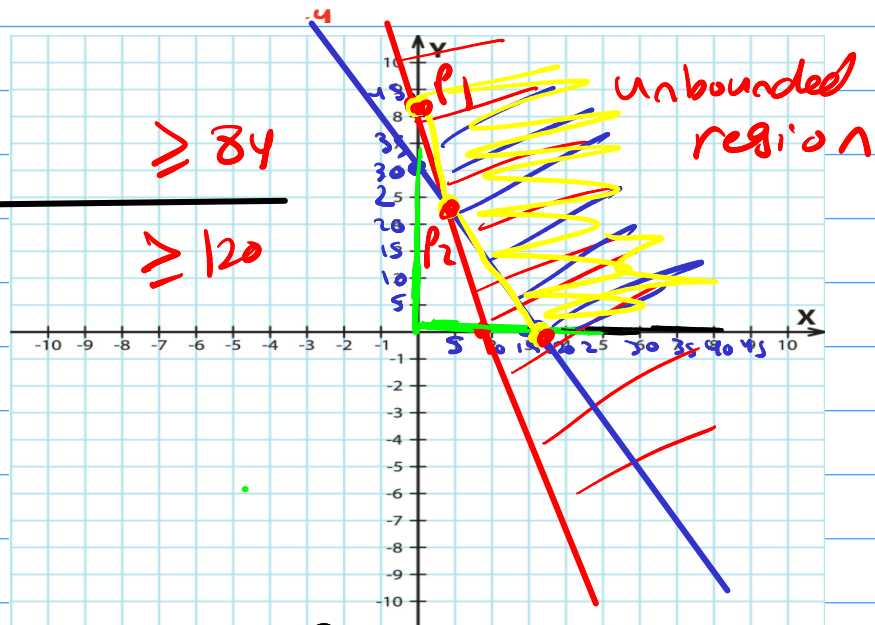
$$(0, 30) \quad (15, 0)$$

$$4y = 120 \quad 8x = 120$$

$$\begin{array}{rcl} -2 \times & 10x + 2y = 84 & \\ & 8x + 4y = 120 & \\ & -20x - 4y = -168 & \end{array} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\}$$

$$-12x = -48$$

$$x = \frac{48}{12} = 4$$



$$Z = 3x + y$$

$$P_1 = (0, 42) \rightarrow 42$$

$$P_2 = (4, 22) \rightarrow 56$$

$$P_3 = (15, 0) \rightarrow 45$$

$$\min Z = 42$$

$$x = 0 \quad y = 42$$

$$10(4) + 2y = 84$$

$$2y = 84 - 40$$

$$y = 22$$

HW.3. A psychologist uses two types of boxes with mice and rats. The amount of time (in minutes) that each mouse and each rat spends in each box per day is given in the table:

	Time		Maximum Time Available Per Day
	Mice	Rats	
Box A	10 min	20 min	400 min
Box B	20 min	10 min	320 min

What is the maximum number of mice and rats that can be used in this experiment?
How may mice and rats produce this maximum?

$$\max Z = X + Y$$

$$10X + 20Y \leq 400$$

$$20X + 10Y \leq 320$$

$$X \geq 0 \quad Y \geq 0$$

$$10X + 20Y = 400$$

$$(0, 20) \quad (40, 0)$$

$$20Y = 400 \quad 10X = 400$$

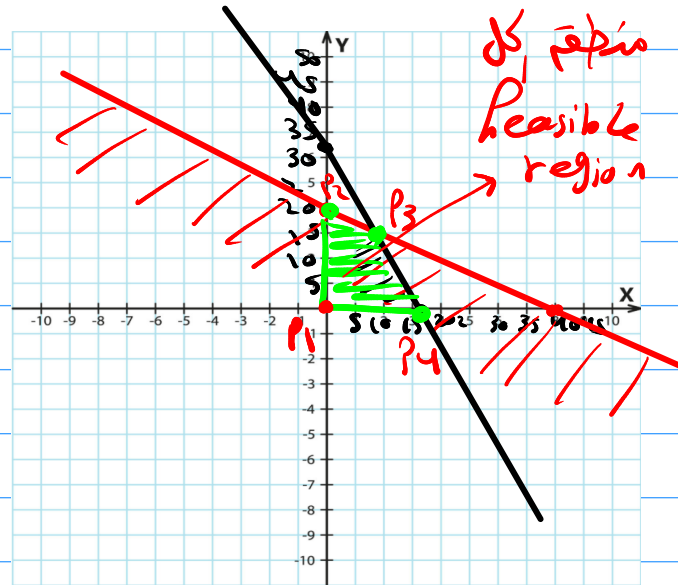
$$20X + 10Y = 320$$

$$(0, 32) \quad (16, 0)$$

$$10Y = 320 \quad 20X = 320$$

$$Y = 32$$

$$X = 16$$



$$Z = X + Y$$

$$P_1 = (0, 0) \rightarrow Z = 0$$

$$P_2 = (0, 20) \rightarrow Z = 20$$

$$P_3 = (8, 16) \rightarrow Z = 24$$

$$P_4 = (16, 0) \rightarrow Z = 16$$

$$\max Z = 24$$

$$X = 8 \quad Y = 16$$

$$② \times 10X + 20Y = 400 \rightarrow Y = 16$$

$$20X + 10Y = 320$$

$$20X - 40Y = -800$$

$$10X + 20(16) = 400$$

$$10X = 400 - 320$$

$$10X = 80$$

$$X = 8$$

$$-30Y = -480$$

$$Y = \frac{480}{30} = 16$$

$$\begin{array}{r} 20 \\ 16 \times \\ \hline 120 \\ 200 + \\ \hline 320 \end{array}$$