EXAMPLE 3 Solving a Formula for a Particular Variable If you deposit a principal P in an account that earns simple interest at an annual rate r, then the amount A in the account after t years is given by $A = P + Prt$. Solve for

(A) r in terms of A, P, and t

(B) P in terms of A, r, and t

$r = ??$

$P = ??$

$$A = P + Prt$$

(A) $r = ?$

$$A = P + Prt$$

$$\frac{A - P}{Pt} = \frac{Prt}{Pt}$$

$$\boxed{r = \frac{A - P}{Pt}}$$

(B) P

$$A = P + Prt$$

$$\frac{A}{(1+rt)} = \frac{P(1+rt)}{(1+rt)}$$

$$\boxed{P = \frac{A}{(1+rt)}}$$

EXAMPLE 6 Diet Jasmine wants to use milk and orange juice to increase the amount of calcium and vitamin A in her daily diet. An ounce of milk contains 37 milligrams of calcium and 57 micrograms* of vitamin A. An ounce of orange juice contains 5 milligrams of calcium and 65 micrograms of vitamin A. How many ounces of milk and orange juice should Jasmine drink each day to provide exactly 500 milligrams of calcium and 1,200 micrograms of vitamin A?

Remark: *A microgram (μg) is one millionth (10^{-6}) of a gram.

	Milk (x)	orange juice (y)	
Calcium	37	5	= 500
Vitamin A	57	65	= 1200

$-13x \quad 37x + 5y = 500 \quad -(1)$

$57x + 65y = 1200 \quad -(2)$

$-481x - 65y = -6500$

$57x + 65y = 1200 \quad (2)$

$(-481 + 57)x = -6500 + 1200$

$-424x = -5300$

$\frac{-424}{-424}x = \frac{-5300}{-424}$

$x = \frac{5300}{424}$

$37x + 5y = 500$

$\frac{37}{1} \cdot \frac{5300}{424} + 5y = 500$

$5y = 500 - \frac{37 \cdot 5300}{424}$

$5y = 500 - \frac{196100}{424}$

37 × 5300 = 196100

37 × 3 = 111, 37 × 7 = 259, 111 + 259 = 370, 370 + 5000 = 5370

37 × 13 = 481, 481 × 100 = 48100, 481 × 3 = 1443, 48100 + 1443 = 49543

500 × 13 = 6500

$y = \frac{500 - \frac{198100}{424}}{5}$ $\quad \frac{198100}{}$

EXAMPLE 6 Purchasing A company that rents small moving trucks wants to purchase 25 trucks with a combined capacity of 28,000 cubic feet. Three different types of trucks are available: a 10-foot truck with a capacity of 350 cubic feet, a 14-foot truck with a capacity of 700 cubic feet, and a 24-foot truck with a capacity of 1,400 cubic feet. How many of each type of truck should the company purchase?

$x_1 =$ 10 Foot Truck $\quad x_1 + x_2 + x_3 = 25$

$x_2 \Rightarrow$ 14 Foot Truck $\quad 350x_1 + 700x_2 + 1400x_3 = 28000$

$x_3 \Rightarrow$ 24 Foot truck $\quad x_1, x_2, x_3 \geq 0$

3 = عدد المتغيرات $\quad$ 2 = عدد المعادلات

infinity many solutions

$$\begin{bmatrix} 1 & 1 & 1 \\ 350 & 700 & 1400 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 25 \\ 28000 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 25 \\ 350 & 700 & 1400 & 28000 \end{bmatrix}$$

-350

$-350R_1 + R_2$

$$\begin{bmatrix} 1 & 1 & 1 & 25 \\ 0 & 350 & 1050 & 19250 \end{bmatrix}$$

-350
25 ×
1750
$7000+$
-8750

19250
350

$\frac{R_2}{350}$

$$\begin{bmatrix} 1 & 1 & 1 & 25 \\ 0 & 1 & 3 & 55 \end{bmatrix}$$

$-R_2 + R_1$

$$\begin{bmatrix} 1 & 0 & -2 & -30 \\ 0 & 1 & 3 & 55 \end{bmatrix} \Rightarrow$$

نرمز لمتغير احد المتغيرات

$$x_3 = t$$

$$x_2 + 3x_3 = 55$$

$$x_2 = 55 - 3t$$

$$x_2 = 55 - 3x_3$$

$$x_1 = 30 + 2t$$

$$x_1 + -2x_3 = 30$$

$$x_1 = 30 + 2x_3$$

Solution

$$x_1 = 30 + 2t$$

$$x_2 = 55 - 3t$$

$$x_3 = t$$

Solution

$t = 0$

$x_1 = 30$

$x_2 = 55$

$x_3 = 0$

$t = 1$

$x_1 = 32$

$x_2 = 52$

$x_3 = 1$

A corporation wants to lease a fleet of 11 buses with a combined carrying capacity of 175 passengers. The three available types of buses carry 11, 14 and 17 passengers, respectively. How many of each type of bus should be leased?

$X_1 \Longrightarrow$ (11 passengers)

$X_2 \Longrightarrow$ (14 passengers)

$X_3 \Longrightarrow$ (17 passengers)

$$X_1 + X_2 + X_3 = 11$$

$$11X_1 + 14X_2 + 17X_3 = 175$$

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$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 11 \\ 11 & 14 & 17 & 175 \end{array}\right]$$

augmented matrix

$-11R_1 + R_2$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 11 \\ 0 & 3 & 6 & 54 \end{array}\right]$$

$-11 \times 11 = -121$

$-121 + 175 = 54$

$R_2/3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 11 \\ 0 & 1 & 2 & 18 \end{array}\right]$$

$3 \times 18 = 54$

$-R_2 + R_1$

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & -7 \\ 0 & 1 & 2 & 18 \end{array}\right] \Longrightarrow$$

$x_3 = t$

$x_2 + 2x_3 = 18$

$x_2 = 18 - 2x_3$

$x_2 = 18 - 2t$

$x_1 + (-x_3) = -7$

$x_1 = -7 + x_3$

$x_3 = t$

$x_2 = 18 - 2t$

$x_1 = -7 + t$

Solution

$x_1 = -7 + t$

$x_2 = 18 - 2t$

$x_3 = t$

$t = 9$

$x_1 = 2$
$x_2 = 0$
$x_3 = 9$

$t = 8$

$x_1 = 1$
$x_2 = 2$
$x_3 = 8$