

6. (15 points) Solve by Gauss-Jordan elimination the following system:

$$x + y + z = 1$$

$$2x + y - 3z = 4$$

$$5x + 4y = 7$$

augmented matrix

$$[A : B] \rightarrow \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 1 & -3 & 4 \\ 5 & 4 & 0 & 7 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$-2R_1 + R_2$$

$$-2(1) + 2 = 0$$

$$-2(1) + 1 = -1$$

$$-2(1) + -3 = -5$$

$$-2(1) + 4 = 2$$

$$-5R_1 + R_3$$

$$-5(1) + 5 = 0$$

$$-5(1) + 4 = -1$$

$$-5(1) + 0 = -5$$

$$-5(1) + 7 = 2$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -1 & -5 & 2 \\ 0 & -1 & -5 & 2 \end{bmatrix}$$

$$R_2 \div -1$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 5 & -2 \\ 0 & -1 & -5 & 2 \end{bmatrix}$$

$$-R_2 + R_1$$

$$+R_2 + R_3$$

$$\rightarrow \begin{bmatrix} 1 & 0 & -4 & 3 \\ 0 & 1 & 5 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

infinitely many solution

حلول غير محدودة، كل

$$z = t$$

$$0x + 1y + 5z = -2$$

$$y = -2 - 5z$$

$$y + 5z = -2$$

$$y = -2 - 5z \quad z = t$$

$$x = 3 + 4z$$

$$x - 4z = 3$$

$$x = 3 + 4z$$

$$z = t$$

Solution

$$x = 3 + 4t$$

$$y = -2 - 5t$$

$$z = t$$

$$at = t = 0$$

$$x = 3$$

$$y = -2$$

$$z = 0$$

$$t = 1$$

$$x = 7$$

$$y = -7$$

$$z = 1$$

Solve by Gauss-Jordan elimination the following system:

$$3x + 2y - 5z = 4$$

$$x - y - 3z = 3$$

$$4x + y - 8z = 11$$

$$\begin{bmatrix} 3 & 2 & -5 & | & 4 \\ 1 & -1 & -3 & | & 3 \\ 4 & 1 & -8 & | & 11 \end{bmatrix} \xRightarrow{R_1 \leftrightarrow R_2} \begin{bmatrix} 1 & -1 & -3 & | & 3 \\ 3 & 2 & -5 & | & 4 \\ 4 & 1 & -8 & | & 11 \end{bmatrix}$$

$$-3R_1 + R_2$$

$$-4R_1 + R_3$$

$$-3(1) + 3 = 0$$

$$-3(-1) + 2 = 5$$

$$-3(-3) + -5 = 4$$

$$-3(3) + 4 = -5$$

$$\begin{bmatrix} 1 & -1 & -3 & | & 3 \\ 0 & 5 & 4 & | & -5 \\ 0 & 5 & 4 & | & -1 \end{bmatrix}$$

$$-4(1) + 4 = 0$$

$$-4(-1) + 1 = 5$$

$$-4(-3) + -8 = 4$$

$$-4(3) + 11 = -1$$

$$-R_2 + R_3$$

$$\begin{bmatrix} 1 & -1 & -3 & | & 3 \\ 0 & 5 & 4 & | & -5 \\ 0 & 0 & 0 & | & 4 \end{bmatrix}$$

$$0 \neq 4$$

No solution
system

Solve the following system of linear equations by Gauss-Jordan elimination method.

$$\begin{aligned} 2x_1 - 5x_2 - 3x_3 &= 7 \\ -4x_1 + 10x_2 + 2x_3 &= 6 \\ 6x_1 - 15x_2 - x_3 &= -19 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 2 & -5 & -3 & 7 \\ -4 & 10 & 2 & 6 \\ 6 & -15 & -1 & -19 \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} 2 & -5 & -3 & 7 \\ 0 & 0 & -4 & 20 \\ 6 & 0 & 8 & -40 \end{array} \right]$$

$$2R_1 + R_2$$

$$-3R_1 + R_3$$

$$2R_2 + R_3$$

$$2(2) + (-4) = 0$$

$$2(-5) + 10 = 0$$

$$2(-3) + 2 = -4$$

$$2(7) + 6 = 20$$

$$-3(2) + 6 = 0$$

$$-3(-5) + (-15) = 0$$

$$-3(-3) + (-1) = 8$$

$$-3(7) + (-19) = -40$$

$$\rightarrow \left[\begin{array}{ccc|c} 2 & -5 & -3 & 7 \\ 0 & 0 & -4 & 20 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

Infinity many solution

جواب بی نهایت

$$-4x_3 = 20$$

$$x_3 = \frac{20}{-4} = -5$$

$$x_3 = -5$$

$$2x_1 - 5x_2 - 3x_3 = 7$$

$$2x_1 - 5x_2 + 15 = 7$$

$$2x_1 - 5x_2 = -8$$

$$x_1 = \frac{-8 + 5t}{2}$$

$$x_2 = t$$

$$x_1 = \frac{-8 + 5t}{2}$$

Solve the following system of equations by Gauss-Jordan elimination.

$$-3x + 2y - 6z = 6$$

$$5x + 7y - 5z = 6$$

$$x + 4y - 2z = 8$$

$$\begin{bmatrix} -3 & 2 & -6 & | & 6 \\ 5 & 7 & -5 & | & 6 \\ 1 & 4 & -2 & | & 8 \end{bmatrix} \quad R_1 \leftrightarrow R_3$$

$$\begin{bmatrix} 1 & 4 & -2 & | & 8 \\ 5 & 7 & -5 & | & 6 \\ -3 & 2 & -6 & | & 6 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 4 & -2 & | & 8 \\ 0 & -13 & 5 & | & -34 \\ 0 & 14 & -12 & | & 30 \end{bmatrix}$$

$$-5R_1 + R_2$$

$$-5(1) + 5 = 0$$

$$-5(4) + 7 = -13$$

$$-5(-2) + -5 = 5$$

$$-5(8) + 6 = -34$$

$$3R_1 + R_3$$

$$3(1) + -3 = 0$$

$$3(4) + 2 = 14$$

$$3(-2) + -6 = -12$$

$$3(8) + 6 =$$

$$\begin{bmatrix} 1 & 4 & -2 & | & 8 \\ 0 & 1 & \frac{5}{13} & | & \frac{34}{13} \\ 0 & 14 & -12 & | & 30 \end{bmatrix} \xrightarrow{R_2 / -13} \begin{bmatrix} 1 & 0 & -\frac{6}{13} & | & -\frac{384}{13} \\ 0 & 1 & \frac{5}{13} & | & \frac{34}{13} \\ 0 & 0 & -\frac{86}{13} & | & -82 \end{bmatrix}$$

$$-4R_2 + R_1$$

$$-4(0) + 1 = 1$$

$$-4(1) + 4 = 0$$

$$+4\left(\frac{5}{13}\right) + -2 = \frac{20}{13} - 2 = \frac{20 - 26}{13} = -\frac{6}{13}$$

$$+14\left(\frac{5}{13}\right) + -12$$

$$\frac{70}{13} + \frac{-12}{1} = \frac{70 - 156}{13}$$

$$-4(8) + \frac{34}{13} =$$

$$-32 + \frac{34}{13} = \frac{-416 + 34}{13} = \frac{-382}{13} + 32$$

$$R_3 \quad \frac{-86}{13}$$

$$\begin{bmatrix} 1 & 0 & \frac{-6}{13} & | & \frac{-384}{13} \\ 0 & 1 & \frac{5}{13} & | & \frac{34}{13} \\ 0 & 0 & 1 & | & \frac{533}{43} \end{bmatrix}$$

$$\frac{+82}{\cancel{+86} \atop 13} = \frac{82 \times 13}{86} = \frac{41 \times 13}{43} = \frac{533}{43}$$

$$\frac{5}{13} R_3 + R_2$$

$$\frac{6}{13} R_3 + R_1$$

$$\frac{5}{13} \left(\frac{533}{43} \right) + \frac{34}{13}$$

$$\begin{bmatrix} 1 & 0 & 0 & | & \frac{-13314}{559} \\ 0 & 1 & 0 & | & \frac{4127}{559} \\ 0 & 0 & 1 & | & \frac{533}{43} \end{bmatrix}$$

$$\frac{5 \times 533}{13 \times 43} + \frac{34}{13} = \frac{2665}{559} + \frac{34}{13}$$

$$\begin{array}{r} 1 \quad 43 \\ 533 \quad 13 \\ 5 \times \\ \hline 2665 \quad 430 + \\ \hline 559 \end{array}$$

$$= \frac{2665 \times 13 + 34(559)}{559 \times 13}$$

$$\boxed{X_1 = \frac{-13314}{559}} \quad \boxed{X_2 = \frac{4127}{559}} \quad \boxed{X_3 = \frac{533}{43}}$$